THE ELEMENTS OF DIRECT CURRENT ELECTRICAL ENGINEERING

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PREFACE

THE main object of the authors of this work has been to bridge the gap existing between the elementary text-books on magnetism and simple current electricity, Ohm's Law, etc., and so many of the more advanced books which deal with the design point of view. Not only the authors, but also many of their friends, have often found difficulty in recommending to students a book which would, without containing many details of a nature more or less superfluous to the purpose, present in a concise form to second-year students at Universities and Technical Schools the theoretical principles underlying Direct Current Electrical Engineering.

No attempt has been made to avoid the use of the Calculus where it simplifies proofs, since it is an essential to all students of engineering; but, in this book, only a very elementary knowledge of that subject is demanded.

With a view to simplification, only typical examples and cases are considered, as, for instance, in the chapter on instruments, since, if these are thoroughly mastered, other types of instruments and machines will be readily understood.

Thanks are due to Messrs. Metropolitan Vickers Electrical Company and the Reason Manufacturing Company for valuable help with drawings and information.

In spite of careful revision, errors easily creep into a work of this nature and, in such a case, the authors will be grateful to have any of these pointed out to them or to receive suggestions as to a second edition, if that should be required.

H. F. T. and G. E. C.

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The Elements of Direct Current Electrical Engineering

CHAPTER I

UNITS, ETC.

§ 1. It is assumed that the student is already familiar with the simple phenomena of electro-magnetism and the elementary definitions, such as unit pole, unit current, etc., but for purposes of revision it would be convenient to summarize some of the salient facts before attempting to apply them to the study of electrical engineering.

The following system of units is called the centimetre-gramme-second system, and is usually denoted by the letters C.G.S.

(*a*) Unit Magnetic Pole is that pole which will exert a force of 1 dyne on a pole of equal strength when the distance between the two is 1 centimetre.

(*b*) Unit Current is the passage of unit quantity of electricity per second, and is that current which, when flowing in a conductor, 1 centimetre long, shaped in the form of an arc of 1 centimetre radius, exerts a force of 1 dyne on unit magnetic pole placed at the centre of the arc; or, if the conductor is a complete circle of 1 centimetre radius, the force at the centre will be 2π dynes.

(*c*) Potential Difference between two points is the work done (ergs) in transferring unit electric charge from one point to another.

Electric Motive Force in a closed circuit is the work done in carrying unit quantity of electricity round the circuit.

1—(5297)

(*d*) Unit Resistance. If unit potential is applied to the ends of a conductor, and unit current is caused to flow, then the conductor is said to have unit resistance.

(*e*) Unit Capacity is the capacity of a body which, when charged to unit potential difference, contains unit quantity of electricity.

§ 2. **Practical Units.**—The above system of units taken as a whole is not entirely suitable for practical use. In order that the numbers involved may be handled conveniently, a unit must be chosen so that it represents a normal value which is not so small, or so large, that it is outside the range of everyday use. The units used in general engineering practice are therefore taken as fractions or multiples of those of the C.G.S. system.

(*a*) Current is one-tenth of the C.G.S. unit of current and is known as the Ampere.

(*b*) Quantity is one-tenth of the C.G.S. unit and is known as the Coulomb.

(*c*) Potential Difference is 10^8 C.G.S. units. This was chosen so that unit P.D. should be equal (approximately) to that of a Daniel Cell.

(*d*) Resistance. The resistance of a conductor which, when a P.D. of 1 volt is placed across its ends, allows a current of 1 ampere to flow. Unit resistance is known as the Ohm.

By adopting the above units for current and potential difference, Ohm's Law $\left[I = \frac{E}{R}\right]$ will hold without the introduction of any constants if the Ohm is equal to 10^9 C.G.S. units.

(*e*) Capacity. The unit is the Farad, which is equal to 10^9 C.S.G. units. (The capacity of the earth is $\frac{7}{10000}$ farad.)

(*f*) Work. The unit is the Joule, which is equal to 10^7 ergs.

(*g*) Power. The unit is the Watt and is equal to 1 joule per second.

1 Horse-power = 746 watts.

§ **3.** A dynamo or motor, or, in fact, almost all electrical machinery when reduced to their simplest elements, consist merely of a conductor moving across a magnetic field. The latter is itself produced by an electric current flowing in a coil round a piece of magnetic material. It is therefore necessary, before considering the details of this machinery, to study the underlying principles in some detail.

§ **4. Magnetic Field Due to Current.**—Consider the simplest possible case of a long straight conductor carrying a current. If this wire is passed through a card on which some iron filings are sprinkled, it is found that the filings will arrange themselves in a series of consecutive rings, the centre of which lies on the conductor. Further, if a suspended magnetic needle is placed in the vicinity of the wire, it is deflected. The current flowing in the wire then creates a magnetic field, its lines of force forming consecutive rings round the wire.

§ **5. Direction of the Field.**—Suppose that a suspended magnetic needle is placed immediately underneath a straight conductor, carrying a current. Before the magnet moves, let its axis, and also that of the conductor, lie in the plane of the paper. It is found that the North Pole will move into the plane of the paper and the South Pole outwards. If the current is flowing in the opposite direction the reverse action will take place.

Further, if the conductor is placed underneath the magnet, the North Pole moves out of the paper for the current flowing from left to right, and in the

opposite direction for a reverse current. It is usual to consider the direction of a magnetic field as the direction in which a free North Pole would move when placed in that field.

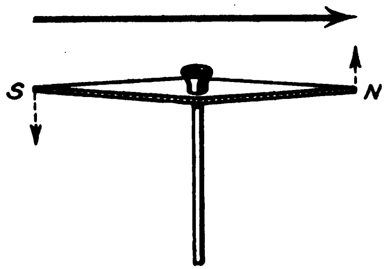

FIG. 1.—DIRECTION OF THE MAGNETIC FIELD DUE TO A CURRENT FLOWING IN A WIRE

RULE. It will be found very useful at a later stage to have some simple rule by which the direction of flow of the current and the direction of the field produced may be readily determined. Consider a right-handed screw travelling in the direction of the

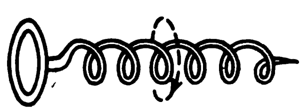

FIG. 2

current, then the lines of magnetic force produced will be in the same direction as that in which the screw turns (*i.e.*, they will be clockwise).

ANOTHER RULE. Suppose that a man is swimming in the direction of the current, facing the wire and on the same side as the magnetic needle, then the North Pole will always be deflected in to his right side.

§ 6. Strength of Magnetic Field due to a Current.—Consider a straight conductor of infinite length, along which a current I is flowing. The current flowing in

an infinitesimal portion may be treated as if it were a small electric charge of magnitude Ids, where I is the current in amperes and ds is the length of the element.

The force then due to this small element at a point O is given by

$$dF = \frac{Ids}{r^2}$$

where r is perpendicular distance from O to the element.

The force due to the whole wire is obviously given by the sum of the forces exerted by all small elements such as ds.

Take a small element BC which subtends a small angle $d\theta$ at O. Let $AO = x$ be the mean distance of the element from O, and let AO be inclined at an angle θ to the perpendicular distance r. Then if the current flowing in BC is I, the effect at O will be the same as that due to a small charge of magnitude $Ixd\theta$, since $xd\theta$ is the component of BC perpendicular to OA. Therefore the force at O due to this element will be given by

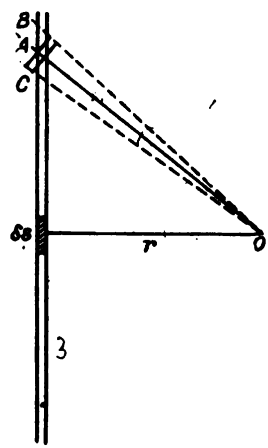

FIG. 3.—MAGNETIC FORCE DUE TO A CURRENT IN A LONG STRAIGHT CONDUCTOR

$$dF = \frac{Ixd\theta}{x^2} = \frac{Id\theta}{x} \quad . \quad . \quad (1)$$

Now $\frac{r}{x} = \cos\theta$. Therefore Equation (1) becomes

$$dF = \frac{I\cos\theta\, d\theta}{r}$$

The effect due to the whole wire is obtained by

integrating this equation between the limits $\theta = -\frac{\pi}{2}$ and $+\frac{\pi}{2}$

$$\therefore \; F = \frac{I}{r}\int_{\theta=-\frac{\pi}{2}}^{\theta=+\frac{\pi}{2}} \cos\theta \, d\theta \quad . \quad . \quad . \quad . \qquad (2)$$

$$= \frac{I}{r}\Big[\sin\theta\Big]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}$$

$$= \frac{2I}{r} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Therefore the force at any point outside the wire varies inversely as the perpendicular distance from the point to the wire.

§ 7. **Force at the Centre of a Circular Coil.**—Let r represent the radius of the coil. Treating the problem in exactly the same manner as that employed in the preceding paragraph, we find that the force at the centre of the coil is

$$F = \frac{2\pi I}{r}$$

where I is the current in the coil. If the coil is made up of N turns, each carrying the current I, then obviously the force at the centre is given by

$$F = \frac{2\pi NI}{r}$$

It will be convenient at this point to consider the units employed in the above equation. If F is in dynes and r in centimetres, the current I will be in C.G.S. units. Hence, if I is expressed in amps. and the other units remain the same,

$$F = \frac{2\pi}{10} \cdot \frac{NI}{r}$$

since an ampere is one-tenth of a C.G.S. unit of current.

The constant $\frac{2\pi}{10}$ we usually replace by its equivalent 1·257, or, for most practical purposes, by 1·26, which is sufficiently accurate.

The quantity NI (*i.e.*, the number of turns multiplied by the current flowing through them) is known as the "ampere turns."

§ 8. **The Solenoid.**—We will next examine the case of a coil whose length is large compared with its cross-sectional area. The coil may be considered as a number

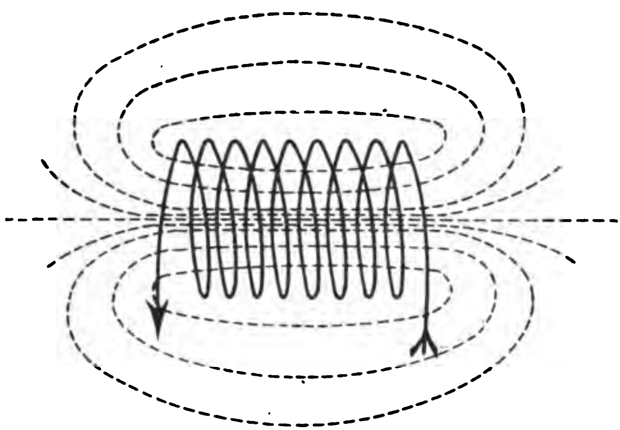

FIG. 4.—MAGNETIC FIELD DUE TO A SOLENOID

of circular conductors placed side by side, each of which sets up its own field.

If a solenoid through which a current is flowing is tested by means of a magnetic needle, it will be found that one end displays North and the other South polarity. Also, if a piece of cardboard sprinkled with iron filings is placed about the coil, the filings will set themselves along the lines of force and, further, these lines of force are exactly similar to those of a

bar magnet. They all enter the coil at the South end and emerge from the North, then they spread out into the surrounding medium and re-enter at the South end. In their passage through the coil they travel along approximately straight lines, except in the immediate neighbourhood of the ends they begin to diverge.

§ 9. Direction of Field Due to Solenoid. — Knowing the direction in which the coil is wound, by using the rule given in (§5), it is an easy matter

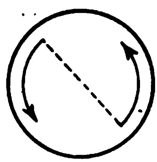

FIG. 5

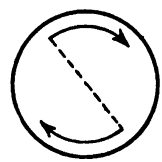

FIG. 6

to deduce the direction in which the lines of force flow. A convenient method of remembering which end of the solenoid exhibits South polarity and which North is the following. If the current is flowing in a clockwise direction the circle (Fig. 6) represents a section through the solenoid, then the South end will be the one nearer to the observer. An easy rule for determining this is to complete a letter S by joining the two arrows which denote the direction of the current. Similarly a current flowing in the counter clockwise direction produces a north polarity nearer to the observer. In this case the letter N may be formed from the two arrows representing the direction of the current.

STRENGTH OF THE FIELD DUE TO A SOLENOID.— We require to find the strength of the field at any point O lying on the axis of the solenoid. Let r represent the radius of the coil. The horizontal force

at any point P due to one turn (AB) which subtends an angle 2θ at O is given by

$$H = \frac{2\pi Ir}{d^2} \sin \theta$$

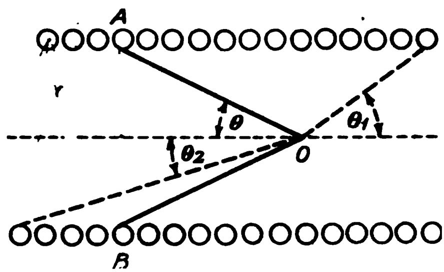

FIG. 7.—MAGNETIC FIELD AT A POINT INSIDE A SOLENOID

where d is the distance AO. (Note that the force is at right angles to AB.) Therefore

$$H = \frac{2\pi I}{r} \sin^3 \theta$$

since $\quad \sin \theta = \frac{r}{d}$.

Suppose that there are N turns per unit length. Therefore in a small length dx these will be $N.dx$ turns. Hence the force due to a small element dx which subtends an angle θ at O is given by

$$H_{(dx)} = \frac{2\pi IN \sin {}^3\theta}{r} . dx$$

Now $\frac{x}{r} = \cot \theta$, therefore $dx = -r \operatorname{cosec} {}^2\theta d\theta$

Hence, $H_{dx} = -2\pi IN \sin \theta d\theta$.

The field at O, due to the whole coil, is given by integrating this expression between the limits

determined by the ends of the coil. Let $2\theta_1$ and $2\theta_2$ be the angles subtended at O by a diameter across each end of the coil respectively. Then

$$H = 2\pi IN\int - \sin\theta d\theta$$

$$= 2\pi IN\ [\cos\theta_1 + \cos\theta_2]$$

at the centre of the coil.

$$H = 4\pi IN \cos\theta_3$$

where θ_3 is the angle subtended at the centre of the coil by a radius at the end.

If the solenoid is very long compared with its diameter θ_3 approximates to zero and $\cos\theta_3$ to unity. The expression for H then becomes

$$H = 4\pi NI$$

If I is expressed in amps.,

$$H = \frac{4\pi}{10}NI$$

Therefore the field strength in dynes $= \frac{4\pi}{10} \cdot \times$ ampere turns per cm. length of coil.

§ 10. Lines of Magnetic Force.—Up to this point we have used lines of force as an aid in expressing the direction of the magnetic force at any point. It will be found useful to use them to express the intensity as well as the direction.

In the simple experiment of sprinkling iron filings on a card surrounding a bar magnet, the filings set themselves along the lines of force and give some idea of the direction of the magnetic force at points in the immediate vicinity of the magnet. Further, it will be noticed that at the pole tips the filings are gathered in thick clusters, while some distance away they are comparatively scattered. We then speak of the lines of force as crowding together at the poles where the

magnetic force is strong and diverging when some distance from the magnet where the force is weaker. If now we could arrange these lines of force so that the number passing through unit areas would represent the force in dynes on unit north pole placed at the centre of the area, we should possess a convenient method of representing the direction and magnitude of the force at any point in a magnetic field. For example, we state that a field strength at a point is 500 lines per sq. cm. This is equivalent to saying that if unit north pole is placed at this point it will experience a force of 500 dynes. It will be necessary, however, to fix this unit area so that the lines of force are perpendicular to it.

As an example, consider two unit north poles 1 cm. apart. The force exerted by one north pole on the other is 1 dyne, therefore we can now fix these hypothetical lines of force radiating from each pole. We have said that the number of lines of force per unit area shall represent the force in dynes. Hence the number of lines of force from one pole passing through an area of 1 sq. cm. round the other shall be one. Therefore since the area of a sphere 1 cm. radius is 4π sq. cms. and one line of force passes through each sq. cm., we can say that unit pole has 4π lines of force radiating from it.

The idea of representing unit magnetic intensity by one line per sq. cm. is quite conventional; for example, it is possible to have a field strength of less than one line per sq. cm. Since a fraction of a line cannot be represented graphically, we could illustrate this field by drawing, say, one line passing through two or more sq. cms.

§ **11. Flux.**—Suppose that we have a uniform field strength H passing normally through a coil whose area is A. Then the total number of lines of force

passing through the coil will be given by HA. This quantity is known as the flux, and is denoted by Φ.

§ 12. **Magnetic Potential and Magneto Motive Force.**—Suppose that unit north pole is moved a short distance dl along any path in a magnetic field. Let the distance dl be so small that the magnetic force may be considered constant over its length. Then the work done is Hdl ergs. H is the magnetic force in dynes and dl is expressed in centimetres. Hence the work done in moving unit north pole over any definite distance between two points is given by the integral

$$\int Hdl,$$

and is defined as the difference in magnetic potential between the two points.

Next consider any line of force, say, round a long straight conductor in which a current i C.G.S. units is flowing. Let the radius of this circular line of force be r cms. Then the force at any point on this circle is given by $H = \frac{2i}{r}$ dynes. Suppose now that we carry unit north pole round the entire circle, then the work done is

$$\frac{2i}{r} \times 2\pi r = 4\pi i \text{ ergs.}$$

This quantity is analogous to the electro-motive force in an electric circuit, and is known as the magneto-motive-force (being usually denoted by the letters M.M.F.). It may be defined as the work (in ergs) done when unit magnetic pole is moved round a magnetic circuit.

Consider the case of a coil of wire made up of N turns each carrying a current I. This is equivalent to a single coil carrying a current NI. The magneto-motive-force round any path linked with this coil is

$4\pi NI$. If I is expressed in C.G.S. units, this quantity will be measured in ergs, but if I is expressed in amps. the magneto-motive-force will be given by $\frac{4\pi}{10} NI$, or, more usually, $1{\cdot}26\ NI$, that is

$$\text{M.M.F.} = 1{\cdot}26 \times \text{Ampere turns of the coil.}$$

§ 13. Reluctance of a Magnetic Circuit.—This quantity may be defined as the ratio of the M.M.F. applied to the circuit, to the flux produced by this M.M.F., that is—

$$R = \frac{\text{M.M.F.}}{\Phi}$$

where R is the reluctance of the circuit and Φ the flux produced.

The M.M.F. may be regarded as the force which is driving the flux round the circuit in exactly the same manner as the electro-motive-force drives a current round an electric circuit.

Reluctance of a magnetic circuit is then analogous to resistance in an electric circuit and will have somewhat similar characteristics. It will be proportional to the length and inversely proportional to the cross-section of the circuit, or

$$R \propto \frac{l}{A}$$

where A is the cross-sectional area and l the length.

R also depends on the material of which the magnetic circuit is made and on the degree of saturation or amount of flux.

This quantity will be dealt with in more detail in Chapter III, in which the properties of magnetic material are examined.

EXAMPLES.

(1) Given a circular coil of 50 turns, and mean radius 5 cms. carrying a current of one-tenth amp. Find the magnetic force at a point whose perpendicular distance to the centre of the coil is 10 cms.

(2) A solenoid, length, 20 cms., diameter, 10 cms. number of turns, 100. Calculate the magnetic force at the centre and at one end of the coil if the exciting current is 2 amps.

(3) Find the number of turns per unit length required on a long solenoid diameter 10 sq. cms. to produce a total flux of lines if the exciting current is limited to 4 amps.

CHAPTER III

INDUCED CURRENTS

§ 1. In 1831 Faraday discovered that currents may be induced in a closed circuit by varying the magnetic field in its vicinity. This great discovery may be demonstrated in the following manner. Consider a bar of magnet N. S. and a coil made up of a number of turns of copper wire. If the magnet is suddenly moved from a position denoted by *ab* to that denoted

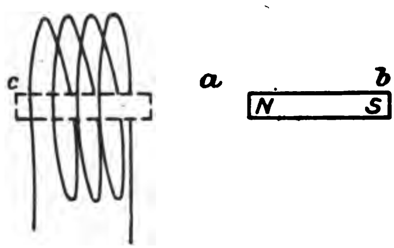

Fig. 8

by *ca*, a momentary current may be observed to flow while the magnet is moving. As soon as the magnet comes to rest, the current dies away to zero. If now the magnet is suddenly withdrawn, a momentary current again flows, but this time in the opposite direction; also, if the magnet is held stationary and the coil is moved, the same effect is obtained. It may further be noted that if the coil is moved slowly, very little current is induced, and the more rapid the movement, the greater the strength of the induced current.

In performing the above experiment, it is found that the current induced when the magnet is entering

the coil is opposite in direction to that when the magnet is leaving.

Take, for simplicity, a single coil of wire and a bar magnet with the North Pole towards the coil. As the magnet is being forced into the coil, the current induced flows in the direction indicated by the arrows (Fig. 9).

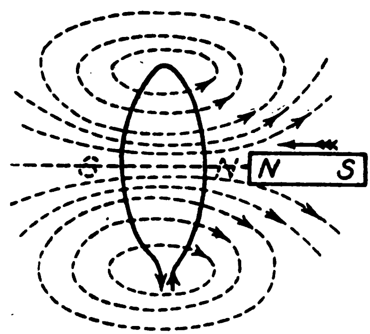

FIG. 9.—DIRECTION OF THE MAGNETIC FIELD DUE TO AN INDUCED CURRENT

Next consider the fields due to the magnet and to the current flowing in the coil. The lines of magnetic force set up by the induced current are in the same direction as those of a magnet with its axis through the centre and at right angles to the coil, its North Pole pointing in the direction of the North Pole of the magnet entering the coil. The field set up by the current obviously tends to repel the bar magnet which is being thrust into the coil.

Similarly when the South Pole of the bar magnet is thrust into the coil, the induced current is in such a direction as would produce a field with its South Pole in the direction of the incoming magnet.

The cases when the magnet is fixed and the coil moving may be worked out by the student.

It is seen from the above experiments that the induced current sets up a field which opposes the motion of the bar magnet, and, further, that the more rapid the motion the stronger the induced current. If the circuit is not closed, no current flows, but a potential difference is set up which would produce a current if the circuit were closed.

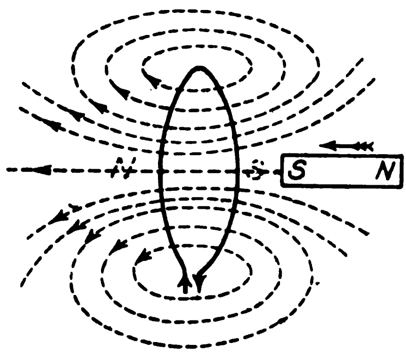

FIG. 10.—DIRECTION OF THE MAGNETIC FIELD DUE TO AN INDUCED CURRENT

These facts may be stated in the following laws—

(1) In all cases of electro-magnetic induction, the induced currents are in such a direction that their reaction tends to oppose the motion which produces them. (This is known as Lenz's Law.)

(2) The E.M.F. induced in a conductor is proportional to the rate at which it cuts the lines of magnetic force ; *e.g.*—

Let a straight wire, length l, be moving at a velocity of v cms. per sec. across a magnetic field strength H. The force on the wire is given by $H.l.i$ where i is the current induced in the wire.

Therefore the work done in 1 sec. $= H.l.i.v$ C.G.S. units.

If e is the electric-motive force induced, the electrical energy produced is ei. If there are no losses, the whole of the work done is transformed into electrical energy and

$$ei = Hliv$$

$$\text{or,} \quad e = Hlv \text{ C.G.S. units} = Hlv\,10^{-8} \text{ volts}$$

(*i.e.*, the E.M.F. induced is equal to the number of lines of force which are cut per second): 1 volt is induced by cutting 10^8 lines per second.

FLEMING'S RULE. It will be found extremely useful when treating the dynamo and motor to have some simple rule connecting the directions—

(1) Of the conductor.
(2) Movement of the conductor.
(3) Of the lines of magnetic force.

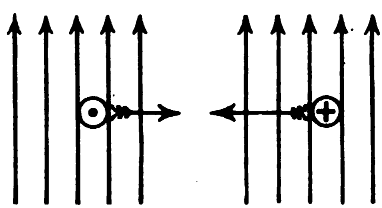

FIG. 11.—ILLUSTRATION OF FLEMING'S RIGHT-HAND RULE

Let the first finger of the right hand point in the direction of the magnetic lines, the thumb represent the direction of motion, then the second finger will represent the direction of the induced E.M.F. This will probably be clear from the following diagrams. The direction of motion is represented by the arrows (Fig. 11).

The vertical lines represent the direction of the magnetic lines of force and ⊕ represents a current flowing downwards, perpendicular to the plane of the paper. ⊙ represents a current in the opposite direction.

§ 2. **Self and Mutual Induction.**—The production of an electro-motive-force may be divided under two headings—

(*a*) When the E.M.F. is induced by the motion of a conductor across a magnetic field ; and

(*b*) When the conductor is stationary and the field moves.

The former is known as a dynamically induced E.M.F. and the latter is known as a statically induced E.M.F.

A simple case of a statically induced E.M.F. may be shown as follows—

Take two coils of wire, *A* and *B*. *B* is connected to a sensitive galvanometer and *A* to some source of current. By suddenly starting or stopping the current in *A*, a current is found to be induced in *B*. It is obvious that this must be so, because a current in *A* sets up a magnetic field and some of its lines of force are linked with *B*. When the current in *A* is suddenly broken or varied, the number of lines of force with *B* are suddenly changed. Therefore an E.M.F. is induced in *B*. This is known as a case of mutual induction and has many practical applications (*e.g.*, transformers).

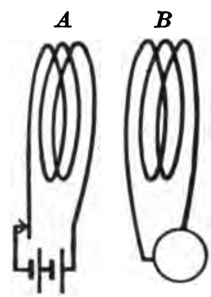

FIG. 12.—MUTUAL INDUCTION BETWEEN TWO COILS

A is usually called the primary coil ; *B* the secondary coil.

Let the two coils be so placed that the whole of the lines of force produced by *A* are linked with *B*. Let S_1 be the number of turns on the primary, and S_2 be the number of turns on the secondary. Then the total number of lines of force or the total flux through *A* due to any current *i* amps. flowing in it, is KS_1i,

where K is a numerical constant depending only on the shape of the coil.*

Hence, if there are S_2 turns on the secondary, the E.M.F. induced is given by

$$- S_2 \times \text{rate of change of flux (C.G.S. units}$$
$$= - S_2 \times \text{rate of change of flux} \times 10^{-8} \text{ volts}$$
$$= - KS_1S_2 \frac{di}{dt} 10^{-8} \text{ volts.}$$

$K\ S_1S_2\ 10^{-8}$ is a constant for the two coils and is known as the coefficient of mutual induction, usually denoted by M. The units being known as "Henries."

§ 3. **Self-Induction.**—It has been shown in the treatment of mutual induction that by varying the current in one coil the flux through an adjacent coil is varied and an E.M.F. induced. It follows, therefore, that if the current in a single coil is increased or diminished, the flux linked with it will be increased or diminished, and this, according to Lenz's Law, will induce an E.M.F. in the coil itself tending to prevent any change of flux. Thus, a coil will act inductively upon itself.

Consider a coil of S turns in which a current of i amps. produces a flux Φ. If the strength of the current is varied, then the E.M.F. induced

$$= - S \times \text{rate of change of flux (C.G.S. units)}$$
$$= - S \times \text{rate of change of flux} \times 10^{-8} \text{ volts}$$

$$E = - S \cdot \frac{d\Phi}{dt} \cdot 10^{-8} \text{ volts}$$

Assuming that the flux is proportional to the current, then if Φ_1 is the flux produced by 1 amp.

$$\Phi = \Phi_1 i$$

Therefore the self-induced E.M.F.

$$E = - S\Phi_1 \cdot \frac{di}{dt} \times 10^{-8} \text{ volts}$$

* For a circular coil containing no iron the flux $= 1 \cdot 26 \cdot S_1 i$.

$S\Phi_1\, 10^{-8}$ is a constant for the coil and is known as its coefficient of self-induction, or its self-inductance, and is usually denoted by L, the unit being the Henry as in mutual induction.

Self-inductance = Number of turns × Flux produced by 1 amp. × 10^{-8}.

Hence,

Volts induced = Henries × Rate of change of current in amp. sec.

$$\text{or,} \qquad E = -L\frac{di}{dt}$$

(*e.g.*, a straight solenoid of length l cms. and cross-section A sq. cms.)

$$\text{Flux per amp.} = \frac{\text{M.M.F.}}{\text{Reluctance}}$$

$$= \frac{1{\cdot}26\,.\,S}{\frac{l}{A}} = \Phi_1$$

Hence the inductance of the solenoid is given by

$$L = S\Phi_1\, 10^{-8} = 1{\cdot}26\,.\,S^2\frac{A}{l}\,10^{-8}$$

For air coils the value of L varies from 1 or 2 millihenries to 1 or 2 henries.

For a small induction coil giving a 2 in. spark, the inductance of the secondary is of the order of 50 henries.

A large induction coil giving a 10 in. spark, 2000 henries.

Shunt field of a dynamo, 1–1000 henries.

Direct current armature, 1/30–50 henries.

§ 4. Rise of Voltage on Breaking an Inductive Circuit.—In circuits of large inductance, such as the field windings of a dynamo or motor, a large rise of voltage may occur if the circuit is broken suddenly.

This rise may be very great, so much so as to produce a breakdown in the insulation of the coils. A typical example will probably make this clear.

A field winding resistance 20 ohms and inductance 12 henries. Voltage, 200. Amps, 10.

Suppose the circuit broken at A and that the break takes place in 1/100 sec.

The average rate of change of current is 1000 amps. per sec.

Average E.M.F. induced

$$= L\frac{di}{dt}$$

$$= 12 \times 1000$$

$$= 12000 \text{ volts.}$$

This would be extremely dangerous, therefore it is obvious that some special precautions must be taken in order to protect the machine from damage.

Sparking Resistance.—The usual method employed is to break the circuit in two stages. A fairly large resistance R is shunted across the switch A, and in this resistance is another switch B (Fig. 13). The switch is so arranged that before A is broken, the resistance R is shunted across A with B closed. Suppose that R is about 180 ohms, then the current in the field coils will now become 1 amp. instead of 10, and therefore when the switch B is opened, the rise of voltage will be 1200 volts. The sparking resistance R must not be too large or, when A is opened, practically no alternative path is offered to the current (*see* later). Also R must not be too small or the current will not be cut down sufficiently. This difficulty of obtaining a suitable resistance is overcome by the next method.

A low resistance with a suitable switch B is connected across the terminals of the field coils (Fig. 14).

With the machine running, B is open and A closed. To break the circuit, B is closed and then A opened. Before B is closed a current of 10 amps. is flowing round the field coils, and when B is closed it is arranged that a current of 1 amp. flows round the battery circuit while now the current of 10 amps. round the field coil

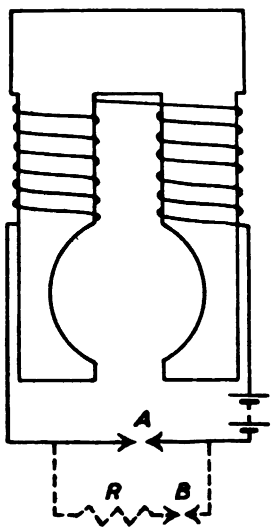

FIG. 13.—ARRANGEMENTS FOR BREAKING A HIGHER INDUCTIVE CIRCUIT

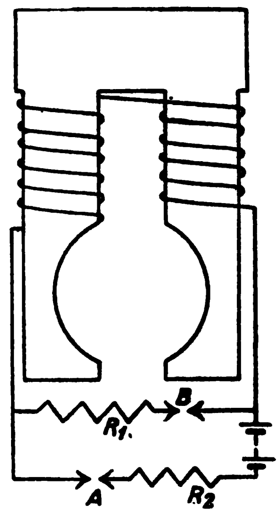

FIG. 14

dies down. On opening A, the current of 1 amp. vanishes almost instantaneously as the circuit is practically non-inductive.

If the sparking resistance R_1 is made as small as possible, when B is closed the battery will be short-circuited. In order to prevent this, a resistance R_2 is introduced before B is closed.

The sequence of events, then, is as follows—

When the machine is running, A is closed and B is open. Then R_2 is introduced and B closed. Then A is opened. This may be done by means of a single switch.

§ 5. **Effect of Short-Circuiting an Inductive Circuit.**—Consider a circuit as shown in Fig. 15. Let its resistance be R ohms and its inductance L henries. Let the coil be short-circuited by a wire of negligible resistance at the same instant as the circuit is broken at A. We wish to find how the current dies down in the coil.

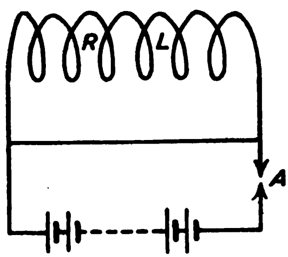

FIG. 15

The current immediately before short-circuit is given by $I = \frac{E}{R}$, where E is the E.M.F. of the battery. Let i represent the current at any instant after the coil has been short-circuited. The E.M.F. driving the current after short-circuit is the induced E.M.F., which is given by $-L\frac{di}{dt}$

Therefore, $$i = \frac{\text{E.M.F.}}{\text{Resistance}} = -\frac{L}{R}\cdot\frac{di}{dt}$$

hence, $$\frac{di}{i} = -\frac{R}{L}\cdot dt$$

Integrating, $$\log i = -\frac{R}{L}\cdot t + C.$$

The value of the constant C is given by the initial conditions when $t = o$, $i = I = \frac{E}{R}$

hence, $$i = \frac{E}{R}\cdot e^{-\frac{R}{L}\cdot t}$$

This may be written

$$i = \frac{E}{R} e^{-\frac{t}{\frac{L}{R}}} = \frac{E}{R} \cdot e^{-\frac{t}{T}}$$

where $T = \frac{L}{R}$ and is a constant for the circuit.

When $t = T \quad i = \frac{1}{e} . I$

T, known as the time constant, is the time taken by the current to fall to $\frac{1}{e}$ th part of its initial value.

$$\frac{1}{e} = \frac{1}{2{\cdot}718} = {\cdot}368$$

We therefore see that the current dies away according to an expotential law.

§ 6. Rise of Current in an Inductive Circuit.—Consider a simple coil, resistance R and inductance L. Let the applied E.M.F. be E and the current at any instant i. Then the opposing E.M.F. due to inductance is $-L\frac{di}{dt}$

The current at any time t is given by

$$i = \frac{E}{R} - \frac{L}{R}\frac{di}{dt}$$

or $$\frac{R}{L} \cdot i + \frac{di}{dt} = \frac{E}{L} \quad . \quad . \quad . \quad . \quad (4)$$

Multiply through by $e^{\frac{R}{L} \cdot t}$

then $$\frac{R}{L} \cdot ie^{\frac{R}{L} \cdot t} + e^{\frac{R}{L} \cdot t} \cdot \frac{di}{dt} = \frac{E}{L} \cdot e^{\frac{R}{L} \cdot t} \qquad (5)$$

which may be written

$$\frac{d}{dt}\left(ie^{\frac{R}{L} \cdot t}\right) = \frac{E}{L} \cdot e^{\frac{R}{L} \cdot t}$$

Integrating, $ie^{\frac{R}{L}\cdot t} = \frac{E}{R}\cdot e^{\frac{R}{L}\cdot t} + C = Ie^{\frac{R}{L}\cdot t} + C$

$$\text{or,} \qquad i = I + Ce^{-\frac{R}{L}\cdot t} \quad . \quad . \quad . \quad (6)$$

The constant C may be obtained from the initial conditions.

When $t = 0$, $i = 0$ $\therefore C = -I$

Hence, current at any instant is given by

$$i = I\left[1 - e^{-\frac{R}{L}\cdot t}\right] \quad . \quad . \quad . \quad (7)$$

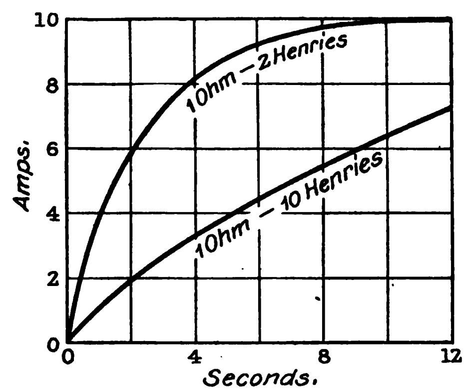

FIG. 16.—RISE OF CURRENT IN AN INDUCTIVE CIRCUIT

As before, $\frac{L}{R}$ is a constant of the circuit and is known as the " time constant."

Equation (7) shows that at any time t the current is smaller than the final value $I = \frac{E}{R}$ by a fraction $e^{-\frac{R}{L}\cdot t}$ When the resistance is large compared with the

inductance, this fraction becomes negligible almost immediately, and the current rises quickly to the full value.

If, on the other hand, L is very large compared with R, the fraction does not become negligible until t has reached some considerable value. This may be demonstrated by plotting the amps.-time curve for two typical circuits in which the full value of the current is the same, but in which the ratios $\frac{R}{L}$ are different (Fig. 16). We thus see that in an inductive circuit it is not possible to start a circuit instantaneously.

Theoretically, the current only reaches its maximum value after infinite time, being always less than its maximum by an amount $Ie^{-\frac{R}{L}\cdot t}$. This item, however, very soon becomes negligible in most practical cases.

§ 7. **Measurement of L. and M.** — The usual practical method of measuring the inductance of a coil is by applying an alternating E.M.F. to it, then measuring the current (I) through it and the potential difference (V) across the ends. In addition to these two quantities it is necessary to know the frequency (n) of the current. The inductance of the coil is given by $\frac{V}{2\pi n I}$ henries, if the resistance of the coil is negligible compared with the inductance.

§ 8. **Ballistic Galvanometer.**—This is a special type of galvanometer which is used in many electrical experiments when it is required to measure currents which flow for a very short time. The instrument may be of the moving coil or magnet type, its chief requirements being a long period of vibration and as little damping effect as possible. These conditions are obtained by giving the moving portions a large moment of inertia and a fine suspension.

We will now consider the case of the moving coil type, since it is the one generally used.

If a steady current (I) flows through the coil of the galvanometer a couple, K_1I, is set up, tending to produce a deflection. Let the deflection be denoted by θ, then the controlling couple called into play is $K_2\theta$. K_1 and K_2 are constants for the instrument and depend on the shape of the coil, the number of

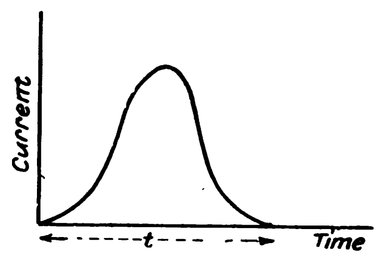

FIG. 17.—CURRENT-TIME CURVE FOR BALLISTIC GALVANOMETER

turns, type of suspension, etc. When the moving portions have arrived at a state of equilibrium,

$$K_1I = K_2\theta$$

or,

$$I = K\theta$$

where $K = \frac{K_2}{K_1}$ a constant for the instrument. Its physical meaning is clearly the value of the current which will produce unit deflection (*i.e.*, a deflection of 1 radian).

If now a transient current is passed through a galvanometer such as the current induced in the Ballistic Test (*see* next chapter) the current flows for a very small interval of time, so small that it is

negligible compared with the periodic time of the moving portions. The current time curve may be represented by a curve of the shape shown in Fig. 17.

If t represents the duration of the current, and i the value of the current at any instant, then the total quantity of electricity passing through the instrument is given by

$$Q = \int_o^t idt.$$

The deflecting couple at any instant $= K_1 i$.

Therefore the impulsive couple

$$= K_1 \int^t idt$$

$$= K_1 Q$$

Let ω denote the initial angular velocity imparted to the moving portions, then, if $\underline{\overline{K}}$ denotes the moment of inertia of the latter, the angular momentum is $\underline{\overline{K}}\,\omega$, and this is equal to the impulsive couple $K_1 Q$.

At the end of the first fling, the kinetic energy $\frac{1}{2}K\omega^2$ has been absorbed, and if θ is the deflection, the work done by the controlling couple is $\frac{1}{2}K_2\theta^2$

$$\text{Therefore,} \quad \tfrac{1}{2}\underline{K}\omega^2 = \tfrac{1}{2}K_2\theta^2$$

$$\omega = \theta\sqrt{\frac{K_2}{\underline{\overline{K}}}}$$

$$\text{but} \quad K_1 Q = \underline{\overline{K}}\omega$$

$$\text{Therefore,} \quad Q = \theta\sqrt{\frac{\underline{\overline{K}}\,K_2}{K_1}} \quad . \quad . \quad . \quad (8)$$

Let T represent the periodic time of the galvanometer.

$$T = 2\pi\sqrt{\frac{\underline{\underline{K}}}{\text{Couple per radian}}}$$

$$= 2\pi\sqrt{\frac{\underline{\underline{K}}}{K_2}}$$

or, $$\frac{K_2 T}{2\pi} = \sqrt{\underline{\underline{K}} \cdot K_2}$$

Substituting in (8) we have

$$Q = \frac{K_2 T}{K_1 2\pi} \cdot \theta = Z\theta$$

Z is the constant of the instrument, since it depends only on the constants K_1, K_2, T.

Hence, the quantity of electricity passing through the galvanometer is proportional to the deflection. The only condition imposed is that the duration of the current must be small compared with the periodic time of the galvanometer.

Before proceeding to the next chapter, the following important theorem should be understood, since the use of the ballistic galvanometer depends largely upon it.

Consider a closed coil of S turns having a resistance R. Suppose there be a certain flux linked with the coil and that it changes by an amount Φ in a short time t. (N.B.—Φ is the *change* and not the total flux.) Then the average induced E.M.F. in each turn will be $\frac{\Phi}{t}$, and so the total E.M.F. will be $\frac{S\Phi}{t}$. This will cause a current $\frac{S\Phi}{tR}$ to flow, and as the time is t, the total quantity of electricity will be $\frac{S \, . \, \Phi}{t \, . \, R} \times t = \frac{S\Phi}{R}$.

Hence, we may say that, when a flux linked with a closed electrical circuit changes, the quantity of electricity which will flow round the circuit is equal to the *change* of flux multiplied by the number of turns divided by the resistance of the circuit.

Examples

(1) A coil of wire is made up of 40 turns, mean diameter 20 cms. and total resistance 2 ohms. It is placed in a magnetic field of 300 lines per sq. cmm.

Calculate the quantity of electricity which will flow round the coil if the field is suddenly reversed.

(2) A circular coil of 50 turns of wire, mean diameter 40 cms. is rotated about a diameter at right angles to a magnetic field of 300 lines per sq. cmm.

If the speed of rotation is 20 revs. per sec., calculate the maximum instantaneous E.M.F. induced.

(3) The shunt field coils of a dynamo have a resistance of 50 ohms and an inductance of 100 henries. They are connected to a battery giving 100 volts. Find the value of the current half a second after the circuit has been closed.

(4) A coil of 50 turns, total length 50 cms., and mean cross-sectional area 10 sq. cms. has a steady current of 10 amps. flowing through it. If the circuit is broken in one-fiftieth of a second, find the rise in voltage across the ends of the coil.

CHAPTER III

ELECTRO MAGNETISM

§ **1.** So far, the magnetic force due to a current flowing in a wire, has been treated without taking into consideration any properties of the medium surrounding it.

The student is probably well acquainted with the fact that when a piece of unmagnetized iron is placed in conjunction with a bar magnet the former will itself exhibit magnetic properties while in this position. Magnetism is said to be induced in the unmagnetized iron. As soon as the iron is removed from the vicinity of the field of the permanent magnet it loses all, or nearly all (*see* later), of its magnetic properties and returns to its normal unmagnetized state. That is to say, the soft iron is only magnetized while in the field of the permanent magnet. Similarly, when a soft iron core is placed inside a solenoid round which a current is flowing, the iron at once assumes all the properties of a permanent magnet to a much greater degree than does the solenoid by itself. Thus, if field strength of the solenoid is H without the iron core, the value of H is greatly increased by the introduction of the core (*i.e.*, new lines of force are brought into evidence). Let the field strength be now represented by B where $B > H$. That is to say, without the iron core there were H lines per sq. cm., and now, due to the presence of the iron, the number of lines of force has been increased to B. H is usually called the Magnetizing Force and B the Magnetic Induction.

§ **2. Permeability.**—Consider a solenoid round which a constant current is flowing. Let the field strength be H when the core contains air only. Now, if various

pieces of iron having the same dimensions but varying in physical and chemical properties, say, from a hard steel to a soft iron, are placed in the solenoid in succession, it will be found that the magnetic induction (B) produced is not the same for each specimen, but will vary between quite considerable limits. The ratio $\frac{B}{H}$, *i.e.*,

$$\frac{\text{Total number of lines after iron is introduced}}{\text{Total number of lines before iron is introduced}}$$

is known as the *Permeability* of the iron in question, and is denoted by μ.

It is obvious that for air and any non-magnetic material the permeability is always unity, since in these cases H is always equal to B.

If, now, the current in the solenoid is increased to twice its original value, with an air core the intensity of the field produced will be $2H$. If, however, the solenoid has an iron core, experiments show that the value of the induction is not usually doubled. The relations between H, B, and μ are not simple, and since nearly all electrical machinery requires the use of iron, magnetized by means of an electric current, it is most important that these relations should be known accurately.

§ 3. **Relation Between B and H.**—There are two methods by which the induction (B) produced in a magnetic substance by a magnetizing force H may be determined.

The first follows an elementary experiment in which the moments of bar magnets are compared. The specimen under test is wound with insulated wire through which a continuous current flows. The intensity of magnetization produced is measured by the deflection of a magnetic needle suspended at a known

distance. This method is both inconvenient and inaccurate. It can only be applied when the length of the specimen is very great compared with its cross-section, because of the disturbing effect of the ends. Further, in order to test a given piece of metal it is not usually convenient to take specimens which would necessitate mechanical treatment to bring them to the required shape. This would alter the magnetic properties of the specimen, which would then bear no relation to those of the original.

§ **4.** The second and most usual method is by means of the Ballistic Galvanometer.

It has been shown in the previous chapter that the ballistic galvanometer measures the quantity of electricity which is passed through it, and that it may be used to measure change of flux. If, then, a ring of metal to be tested is wound with primary and secondary coils, the magnetizing force may be calculated, knowing the particulars of the primary coil and the current through it. If the secondary is connected to the ballistic galvanometer and the primary current changed, the change of flux may be measured, if the constant of the instrument is known. Certain precautions must be taken. The primary must be wound so that the ring under test is completely covered by the coil, in order to prevent flux leakage. The secondary need not necessarily cover the whole coil. A diagram of the apparatus is shown in Fig. 18.

A battery is connected through a reversing switch to the primary, which is in series with an ammeter and resistance. The secondary is connected through a resistance to the ballistic galvanometer.

The test is started with the ring completely demagnetized (*see* later). A small current is then allowed to flow through the primary. Next, a resistance is suddenly cut out of the primary. H is therefore

increased, and the corresponding increase of B produces a deflection of the galvanometer. The increased value of the current and the deflection are measured. This process is repeated for increasing values of the primary current.

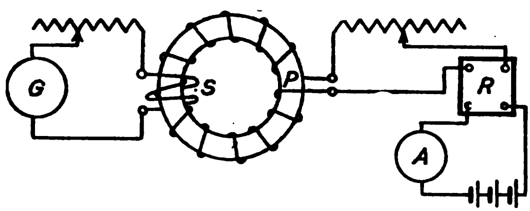

FIG. 18.—APPARATUS FOR THE BALLISTIC TEST

A. Ammeter
R. Reversing switch
P. Primary coil
S. Secondary coil
G. Ballistic galvanometer

For each step—

(1) The magnetizing force $1{\cdot}26 \, . \dfrac{SI}{l}$, and

(2) The magnetic induction B produced, may be calculated.

$$B = \frac{KR_2}{AS_2}\theta$$

where K is the constant of the ballistic galvanometer, R_2 is the total resistance of the secondary circuit, A the area, S_2 the number of turns on the secondary coil.

B-H CURVE.—It is now possible to determine the relation between B, H, and μ, for any given sample of iron.

If the results obtained are plotted, a curve having a shape similar to Fig. 19 is obtained.

The magnetizing force is plotted along the horizontal axis, the corresponding values of the induction along the vertical axis.

Commencing with the iron completely demagnetized, the curve shows that at first the induction is proportional to the magnetizing force. In other words,

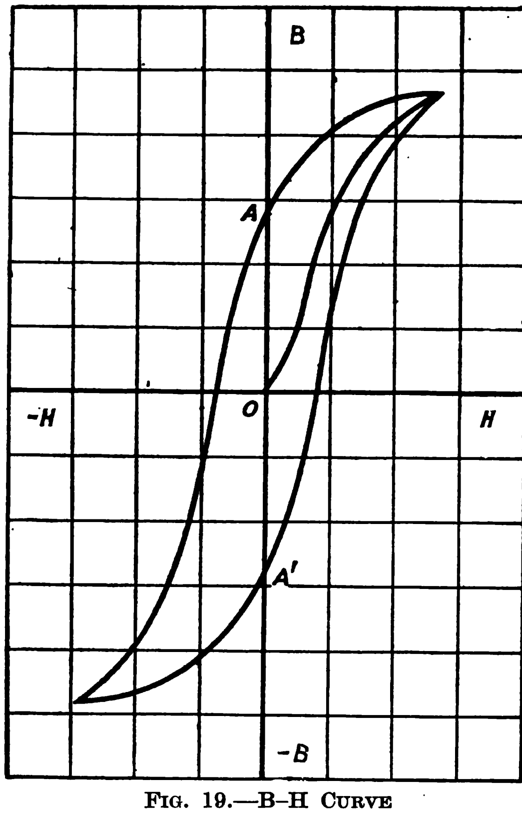

FIG. 19.—B–H CURVE

when H is very small the induction is exactly the same as if the coil had an air core instead of iron. Very soon, however, when H is increased by a small amount, the value of B increases enormously, shown by the

curve rising steeply. Then the curve commences to flatten until the increase in B is again equal to the increase in H. At this point the iron is said to be saturated.

If now the current is decreased in steps, a curve of similar shape is obtained. It does not coincide with the curve obtained when H is increasing, but lies above it; that is to say, the value of B for a corresponding value of H is greater when H is decreasing than when increasing. Finally, when H is reduced to zero, it is found that B has a considerable positive value denoted by OA. At this point the current is reversed, and we have a positive value of B with a negative value of H. The magnetizing force is too weak to overcome the residual magnetism in the iron. As H is increased in the negative direction, B passes through zero, becomes negative, and increases rapidly until it reaches its maximum value, exactly as in the case with the current in the positive direction. The current is now reduced in stages to zero when the value of B is given by OA'. The battery is now reversed, and the current increased by steps until the iron again becomes saturated.

If the cycle is repeated, it will be found that the curve will be retraced if the direction of the cycle remains the same.

This curve is typical of all kinds of iron and steel. The actual values of B corresponding to given values of H vary very considerably for, say, cast iron and wrought iron; further, different specimens of wrought iron will also show considerable variations. The curves shown in Fig. 20 give average values for various types of iron and steel.

μ–H CURVE.—From the results obtained in the above experiment, the values of μ corresponding to any value of H may be determined. It is obvious

from the curves that μ increases rapidly with H until it reaches a maximum at the point where the B–H curve flattens out ; that is, when the iron is approaching saturation point. From this point onwards

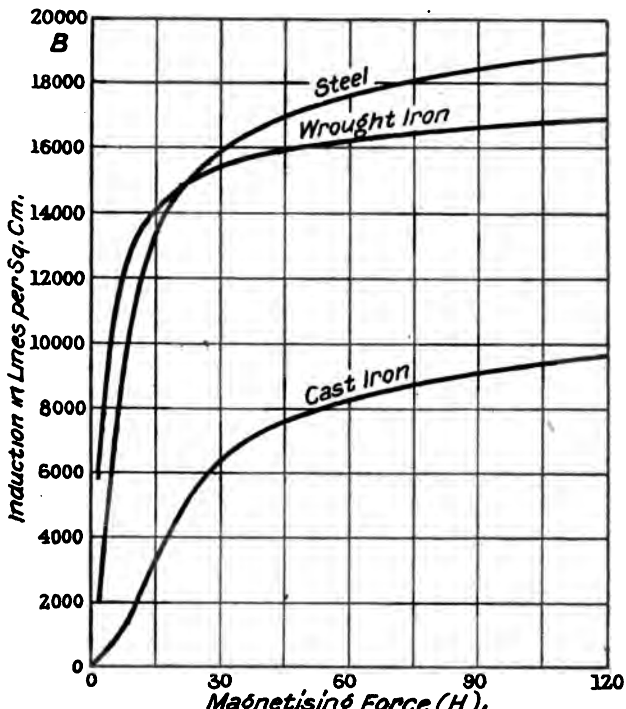

FIG. 20.—TYPICAL B–H CURVES

permeability falls away. The curves (Fig. 21) show typical values for cast and wrought iron and cast steel.

In practice, iron is never worked at or above its saturation point, but always just below. It is obvious that at this point maximum efficiency is obtained ;

that is, maximum induction is obtained for a given number of ampere turns.

§ 5. **Hysteresis.**—It has been shown that the *B–H* curve when *H* is increasing does not follow the same

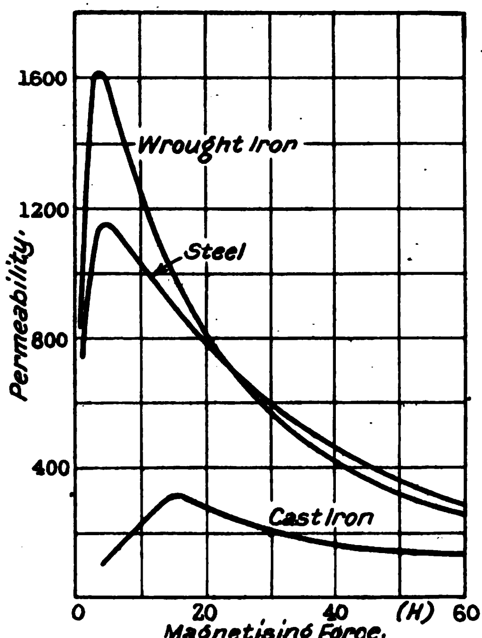

FIG. 21.—TYPICAL μ–H CURVES

line as when *H* is decreasing. When *H* is decreasing *B* lags considerably, so much so that when *H* is reduced from, say, a positive value to zero, the value of *B* does not become zero but retains an appreciable positive value. This phenomenon is known as Magnetic Hysteresis.* The *B–H* loop is usually known as the hysteresis loop.

* (υστερέω to lag).

§ 6. Energy Dissipated in a B-H Cycle.—If an alternating current is supplied to a coil wound on a piece of iron and the energy expended measured, it is found that considerably more power is required than that used in heating the wire (I^2R). This extra energy is necessary to supply that dissipated by magnetic hysteresis and can be shown to be equal to

$$= \frac{\text{Area of one complete } BH \text{ loop}}{4\pi}.$$

in ergs per cc. of iron per cycle.

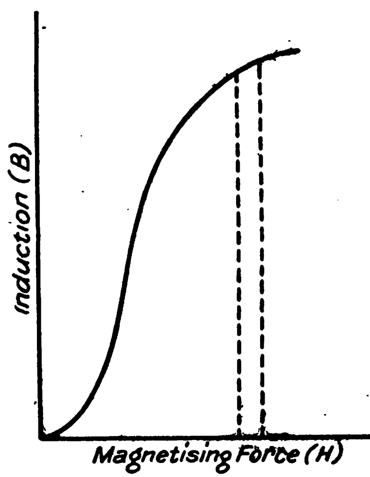

FIG. 22

Let the total number of turns on the coil $= S$.

Let the cross-section of the iron $= A$, and let B and I represent the induction and the current respectively.

Let the value of the induction (B), be increased by

an infinitesimal amount dB in time dt. Then the back E.M.F. induced in the coil is

$$= -\frac{dB}{dt} \times A \times S \quad . \quad . \quad . \quad (9)$$

The power during the time dt due to the current I

$$= \frac{dB}{dt} . A . S . \frac{I}{10} \quad . \quad . \quad . \quad (10)$$

It had been proved that

$$H = \frac{4\pi}{10} \times \frac{I . S}{l} \quad . \quad . \quad . \quad . \quad (11)$$

Substituting in (10) work done in time dt

$$dW = dB . A . l \frac{H}{4\pi} \text{ ergs}$$

Now $A . l$ is the total volume of iron.

$\therefore$ The work done in taking the specimen of iron through one complete cycle is given by

$$\text{Energy expended} = \frac{1}{4\pi} \int H . dB \text{ ergs per cc.}$$

For one complete cycle of the BH curve this integral is obviously the area of the loop.

The energy dissipated in one complete cycle is

$$\frac{\text{Area of } B\text{–}H \text{ loop}}{4\pi} \text{ ergs per cc.}$$

This result is extremely important, since losses due to hysteresis occur in many types of electrical machinery, generators, motors, transformers, etc., and their efficiency will depend to a considerable extent on the hysteresis loss. Further, this energy is dissipated in heat, and the heating of a machine limits its output.

§ 7. **Steinmetz' Law.**—In dynamo design it is important to know how the hysteresis loss varies with different values of the maximum induction. This

question was investigated by Steinmetz, who gave the empirical law—

Hysteresis loss (ergs per cc. per cycle) $= \eta B^{1\cdot 6}_{(\text{max.})}$

η is known as the hysteresis constant. It varies with the kind of iron or steel used, and is of the order of ·0015.

1·6 is known as the Steinmetz exponent.

If the hysteresis loss for any sample of iron is plotted against the corresponding values of maximum

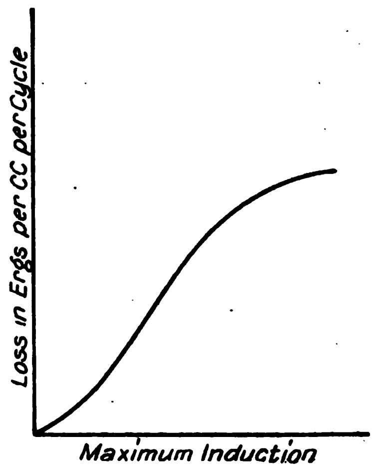

Fig. 23.—Curve Showing the Relation between Hysteresis Loss and Maximum Induction

induction, a curve of the shape shown in Fig. 23 is typical. If, now, the log (loss) are plotted against log B_{max}, a straight line (approx.) is obtained, from which the values of η and the exponent may be found. (*See* Fig. 24.)

The value 1·6 given by Steinmetz is true for values of B up to about 9000. For very low values of B_{max}

a closer approximation is given by B^2_{max}, while for high values of B_{max} the hysteresis loss is practically constant.

§ **8.** To assist the student to understand the principle of hysteresis loss the mechanical analogy may be useful. Consider a metal wire or rubber fibre subjected to varying tension.

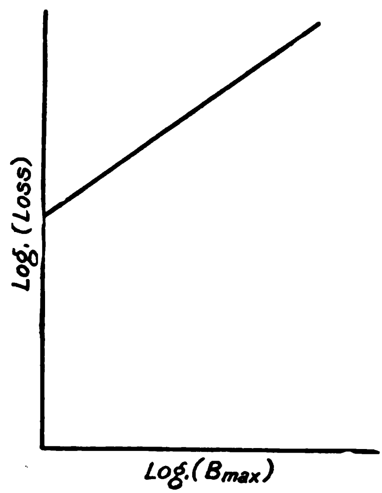

FIG. 24

In Fig. 25 the strain produced is plotted against the stress applied, and it will be seen that the curve for decreasing stress does not coincide with that for increasing stress. *OAB* is the curve when the stress is being applied, and *BCO* when it is being released. The area *OABN* represents the work which is expended, and *OCBN* the work which is recovered. The area *OABC* is therefore work which is lost in the material. This mechanical hysteresis corresponds to the magnetic hysteresis of the *BH* cycle.

§ 9. **Physical Explanation.** — The modern views regarding the foregoing phenomena are founded on the molecular theory. A magnetic body is considered to be made up of molecules, each of which is a magnet.

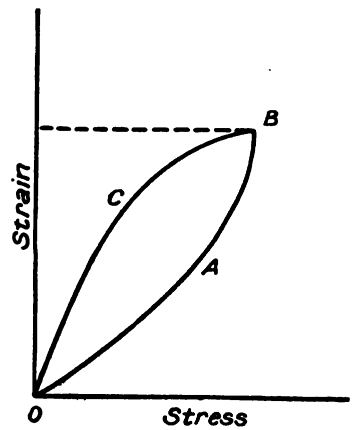

FIG. 25.—CURVE SHOWING THE RELATION BETWEEN MECHANICAL STRESS AND STRAIN

Consider first a non-magnetized piece of iron. There the molecules are arranged in groups so that each group is a closed magnetic circuit. Fig. 26 shows a

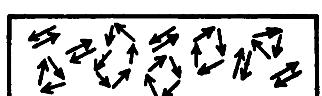

FIG. 26.—MOLECULAR MAGNETS ARRANGED IN CLOSED CHAINS, IN AN UNMAGNETIZED PIECE OF IRON

diagrammatic representation of a number of these groups.

The lines of force of each molecular magnet following

the path of least reluctance never emerge from the chain. Thus, the iron bar displays no external magnetism but will behave exactly as a piece of non-magnetic material. Suppose now that the bar is placed inside a solenoid carrying a very small current. The magnetizing force due to the current will be very weak, so weak that these closed groups of molecular magnets are not broken up. This accounts for the first part of the *BH* curve, where *B* and *H* are proportional, and, further, if the magnetizing force is

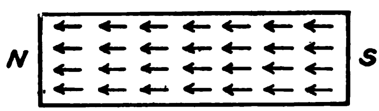

FIG. 27.—ARRANGEMENT OF MOLECULAR MAGNETS IN A MAGNETIZED PIECE OF IRON

removed, the induction *B* also disappears. This follows in the above explanation, since none of the molecular groups are broken by the magnetizing force, or if a few of the groups are broken, they return to their position of equilibrium as soon as the magnetizing force is removed.

When the current in the coil is increased, *H* is increased, so that some of the molecular groups are broken and the magnets arrange themselves so that they lie along the lines of magnetic force (*see* Fig. 27). This corresponds to the steep part of the *BH* curve, since when *H* is increased by a small amount, a large number of these groups, being in a state of unstable equilibrium, are suddenly broken up. The result of this is a large increase in the value of *B*, caused by a very small increase in *H*.

If *H* is now further increased, since the molecular magnets are now nearly all in line, the rate of increase

in B will not be so marked. The B–H curve will flatten out. This, then, gives the explanation of saturation. The molecular magnets are now all in line, and the only increase in B is due to H.

If H is now slowly reduced the magnets tend to remain in their new positions of equilibrium, so that the drop in B will be slight. Thus, when the magnetizing current is reduced to zero, the iron retains a considerable amount of magnetism. When H is next applied in the opposite direction, the magnets become unstable until they are forced to swing round and point once more in the direction of the magnetizing force. When these molecular magnets become unstable they will oscillate rapidly in the manner of an ordinary suspended magnetic needle. This vibrating field will induce eddy currents in the neighbouring iron which will cause an evolution of heat.

The evidence supporting this theory is considerable—

(1) In a piece of iron which is put through a number of cycles its temperature rises. This will be pointed out later as a source of loss in dynamos, etc. Here the armature is put through a continuous cyclical change, and the energy lost due to hysteresis is converted into heat.

(2) If a specimen is placed in (*a*) an alternating field, and (*b*) a rotating field having the same frequency, the iron displays different values for hysteresis loss. This is due to the different rates of oscillation set up by the two fields. The rotating field only produces a steady rotation, while the alternating field sets up violent oscillations, hence the loss in the latter will be considerably greater.

§ **10. Magnetic Testing.**—(*a*) Permeability; (*b*) Hysteresis Loss Tests.

(*a*) BALLISTIC METHOD. This method has been treated in some detail in §4. It is not always

convenient, since the (primary and secondary) coils have to be wound by hand. Sometimes the coil is permanently wound and the test piece is inserted in the form of sheets and the ends jointed. The disadvantage is that the joint tends to change the value of the reluctance.

The most convenient form of test piece would be a short bar on which a primary and secondary could be

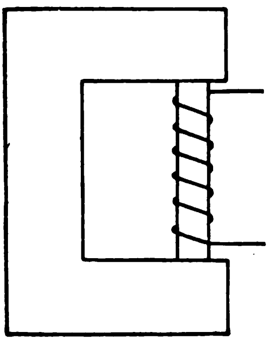

FIG. 28

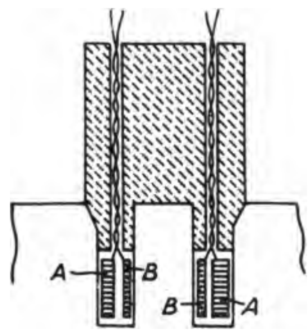

FIG. 29.—THE DRYSDALE'S METHOD OF TESTING A LARGE MASS OF IRON

AÁ. Primary coil
BB. Secondary coil

wound. The difficulty encountered is that the magnetic circuit would be partly in the specimen and partly in air. The ampere turns required for the air are not known, since the mean length of air path is uncertain. This difficulty is obviated by closing the magnetic circuit by means of a soft iron yoke of large cross-section, and therefore of negligible reluctance. Fig. 28 gives a diagrammatic representation of the apparatus.

§ **11. Drysdale Permeameter.**—An apparatus by means of which a large mass of iron may be tested. A cylindrical hole is drilled in the material, leaving a

circular projection at the centre. Round the latter, primary and secondary coils are fitted. The magnetic circuit is closed by means of a plug which fits tightly into the hole. The plug carries the coils and the leads (Fig. 29).

§ **12. Permeability Bridge.**—This method, designed by Ewing, employs a standard bar of iron with which

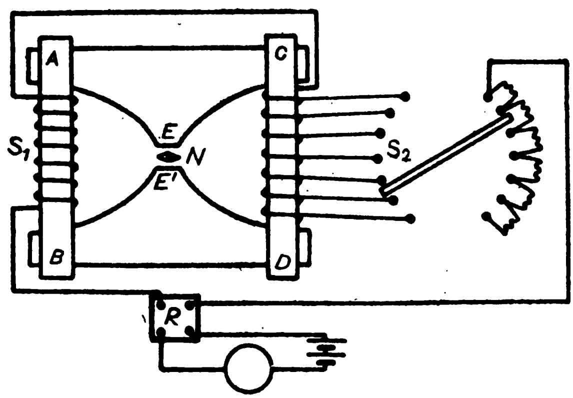

FIG. 30.—EWING'S PERMEABILITY BRIDGE

the specimen under test is compared. The *B–H* curve for the standard bar is known. Fig. 30 shows the arrangement of the apparatus.

AB represents the standard bar and *CD* the bar to be tested. On *AB* is a coil, made up of a fixed number of turns, say, S_1. The coil S_2 wound on *CD* is the series with S_1, and is arranged so that the number of turns on *CD* may be varied, and hence, although the current through the two coils is the same, their respective magnetizing forces may be different. Further, the latter may be readily compared if the number of loops carrying current are known. A refinement is introduced by arranging the dial (varying the S_2

turns on CD) so that as each coil is cut out an equal resistance is introduced, therefore the magnetizing current remains constant.

The two bars are connected by soft wrought iron yokes E and E'. Fitted to each yoke is an arm, and these are brought together so that, in the air gap between their ends, a magnetic needle may be suspended. The needle will indicate if there is any difference between the magnetic states of the two arms.

If the standard and test piece have the same permeability, equal values of B will be produced by the same magnetizing force (*i.e.*, when $S_1 = S_2$) since the same current flows through both coils. If this happened to be the state of affairs there would be no deflection of the magnetic needle. The lines of induction due to the standard would flow by the paths $AEE'B$ and $ACDB$. Those due to the test piece by $DE'EC$ and $DBAC$. When the fluxes through the air gap are equal, there will be no deflection of the needle. Suppose, now, that the induction in CD is greater than in AB, the needle will be deflected. Therefore by varying the S_2 turns, the needle may be brought back to its zero position. That is to say, the magnetizing force acting on CD is varied until the two inductions are again equal.

The magnetizing current on the standard is equal to the current I multiplied by a constant of the instrument (K), and also since the same current flows through both coils, we have

$$\frac{\text{Magnetizing force on test bar}}{\text{Magnetizing force on standard}} = \frac{S_2}{S_1}$$

$$\therefore \text{ Magnetizing force on test bar} = \text{Instrument constant} \times I \times \frac{S_2}{S_1}$$

Now the B–H curve of the standard is known, and

therefore for any value of the magnetizing current the corresponding induction is known. Also from the above data the magnetizing force acting on the test piece can be found, and since an induction equal to that of the standard is produced, the permeability can at once be determined.

It will be noticed that in the diagram a reversing switch R is inserted. This is to eliminate the effect of hysteresis. The current is reversed from time to time, so that errors due to retentivity of the iron may not affect the experiment.

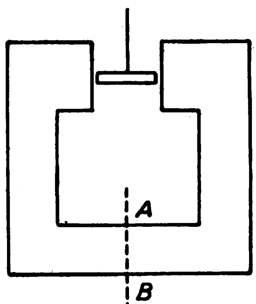

Fig. 31.—Direct Method for the Determination of Hysteresis Loss

§ 13. **Measurement of Hysteresis Loss.**—It has been proved that the area of the BH curve divided by 4π is equal to the energy dissipated per cc. during one magnetic cycle. The hysteresis loss may therefore be calculated from the data obtained in the experiment described in § 4.

A very simple direct method which may be employed is as follows. The test piece in the form of a small disc is suspended by means of a steel wire between the poles of a magnet. The latter may be revolved about an axis AB (*see* Fig. 31). Under these circumstances the magnetism of the test piece is reversed periodically, and therefore, due to hysteresis, a certain amount of work is done. The work done on the test piece in one rotation is equal to its hysteresis loss, and is equal to $2\pi \times$ couple acting on it. This couple may be measured, since it is proportional to the deflection of the test piece.

§ 14. **Ewings' Hysteresis Tester** is similar in principle

to the preceding apparatus, the difference being that the test piece is moved and the magnet is deflected, Fig. 32 shows a diagram of the apparatus.

N S is a permanent magnet to which is attached a pointer *AB* which travels over a scale *C*. The magnet is supported on a knife edge *D*. The oscillation of the

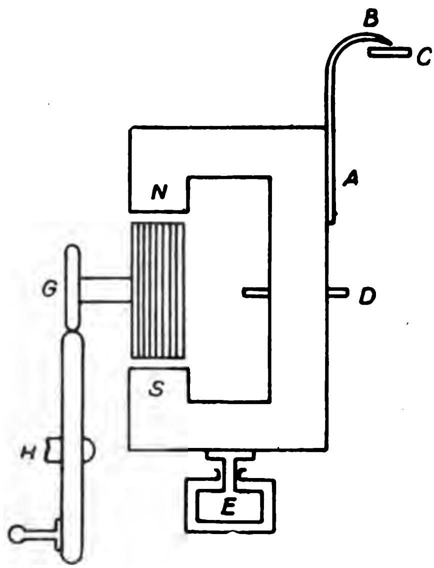

FIG. 32.—EWING'S HYSTERESIS TESTER

magnet is checked by means of a vane *E* working in a small box filled with oil. The test piece is fitted into a holder *F*, which is geared through a small wheel *G* to a hand wheel *H*. In order to calibrate the scale, two standard samples of iron are provided. When the instrument has been adjusted for level, etc., the standard samples are placed in the holder in turn, and the deflection produced is read from the scale. From

these two readings a calibration curve (Fig. 33) may be constructed by drawing a straight line through these two points.

The specimen to be tested is next inserted. From the deflection produced, the hysteresis loss in ergs per cc. per cycle may be read off from the curve. The speed at which the test piece is rotated is immaterial.

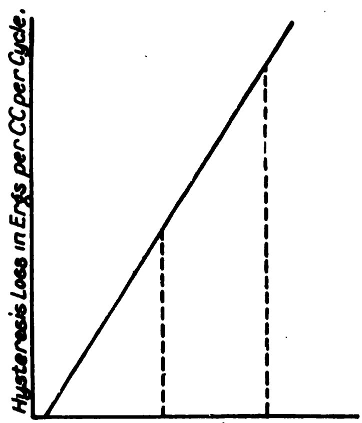

FIG. 33.—CALIBRATION CURVE FOR EWING'S HYSTERESIS TESTER

A fixed amount of work is done in each cycle whatever the speed of rotation, and, therefore, if the speed is high enough not to give the magnet time to move between successive impulses the deflection will remain constant.

It will be noticed in the calibration curve that the line cuts the axis at a point a short distance from the origin. This is due to a certain amount of hysteresis loss taking place at the pole tips, when the test piece is being revolved. Fig. 34 and 35 show the distributions

of the lines of force for the test piece in two positions at right angles, and this variation in the direction of the lines of force will obviously cause a certain amount of loss.

§ 15. **Materials for Magnetic Circuits.**—For an ideal magnetic circuit, the material should have high induction at low values of the magnetizing force.

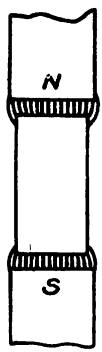

FIG. 34.—DIAGRAM SHOWING THE DISTRIBUTION OF THE LINES OF FORCE FOR TWO POSITIONS OF THE TEST PIECE

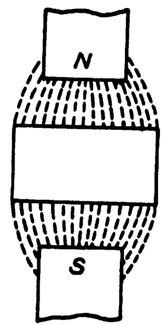

FIG. 35

§ 16. **Reluctance.**—In an electric circuit the resistance is determined by the quantity $\frac{l}{a \cdot K}$ where a is the cross-sectional area, l the length and K the conductivity of the material in question. In a magnetic circuit, *Reluctance* is defined as $\frac{l}{a\mu}$ where μ is the permeability.

The importance of this conception will be seen from the following. Consider an iron ring whose mean length is l, area a, and on which is wound a coil of N

turns. Let a current of I amps. flow through the coil, then the magnetizing force H is

$$1{\cdot}26 \times \frac{N\,I}{l}$$

If Φ represents the flux and B the induction, then

$$\Phi = B \times a = \mu H \,.\, a$$

$$\therefore \quad = \frac{1{\cdot}26\, N\,.\,I}{\frac{l}{\mu\, a}}$$

$1{\cdot}26\, NI$ is the magneto-motive-force. Hence, we have a complete analogy between the electric and magnetic circuits.

Flux in the magnetic circuit corresponds to current in the electric circuit, M.M.F. with E.M.F., and reluctance with resistance.

Consider now a composite magnetic circuit made up of three reluctances in series (*e.g.*, a ring made up of wrought and cast iron and cast steel). Let their lengths be l_1, l_2, l_3, their cross sections a_1, a_2, a_3, their permeability μ_1, μ_2, μ_3 respectively. If the ring is wound with S turns of wire carrying a current I amps.,

$$\text{the flux} \quad \Phi = \frac{1{\cdot}26\, S\, I}{\frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3}} = \frac{1{\cdot}26\, S\, I}{R_1 + R_2 + R_3}$$

Then R_1, R_2, R_3 are their respective reluctances.

If the circuit is divided, with R_1 and R_2 in parallel, as in Fig. 36,

$$\text{total flux} \quad \Phi = \frac{1{\cdot}26\, S\, I}{R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

The difference between an electric and a magnetic circuit, however, is most important. In the former, Ohm's law may be applied directly, but in the latter,

the treatment is not so straightforward. The reason for this is that the permeability is not constant but varies with the magnitude of the magnetizing force.

Consider a circuit made up of different metals, lengths l_1, l_2, l_3; cross-section a_1, a_2, a_3; and permeability u_1, u_2, u_3, respectively.

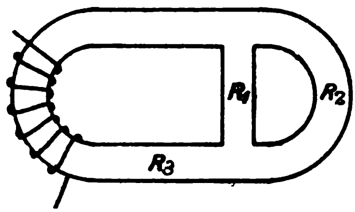

FIG. 36.—A DIVIDED MAGNETIC CIRCUIT

The magneto-motive-force required to send a flux Φ round this circuit is given by

$$\text{M.M.F.} = \Phi \times \text{Reluctance}$$

$$= \Phi\left[\frac{l_1}{\mu a_1} + \frac{l_2}{\mu_3 a_2} + \frac{l_3}{\mu_3 a_3}\right]$$

$\frac{\Phi}{a_1}, \frac{\Phi}{a_2}, \frac{\Phi}{a_3}$, are the values of the induction in each segment, and may therefore be denoted by B_1, B_2, and B_3 respectively.

$$\therefore \quad \text{M.M.F.} = \frac{l_1 B_1}{\mu_1} + \frac{l_2 B_2}{\mu_2} + \frac{l_3 B_3}{\mu_3}$$

but since $\mu = \frac{B}{H}$, we have

$$\text{M.M.F.} = l_1 H_1 + l_2 H_2 + l_3 H_3$$

$$\text{but M.M.F.} = 1{\cdot}26 \times \text{Ampere turns}$$

$$= H_1 l_1 + H_2 l_2 + H_3 l_3$$

$$\therefore \text{Amp. turns} = \frac{1}{1{\cdot}26}\left[H_1 l_1 + H_2 l_2 + H_3 l_3\right]$$

$$= {\cdot}8 \Sigma H l$$

That is to say, the total number of ampere turns required in a composite circuit to produce a given flux, is equal to the sum of the ampere turns required to produce that flux in each section.

§ **17. Leakage.**—In a closed magnetic circuit, such as the one just considered, it would be quite a simple matter to calculate the ampere turns required to produce a given flux. In almost every practical case, however, the effect of leakage has to be taken into account.

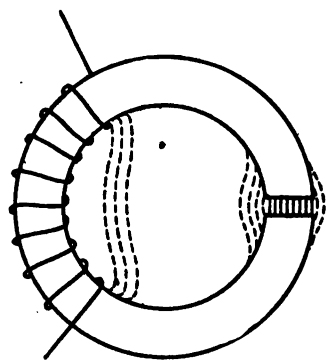

FIG. 37.—MAGNETIC LEAKAGE

A typical example may be taken from the case of a ring which has a short coil over a small portion of its length, and an air gap as shown in Fig. 37.

It is obvious that here the flux does not follow a path which is bounded by the iron, but that lines of force will leak outside the limits of the iron in order to follow the path of least resistance. The amount of leakage will depend on the length of the air gap, the dimensions of the exciting coil, and the degree of saturation of the iron.

In the case of the dynamo the useful flux is that

which passes across the air gap between the pole face and the surface of the armature. This will always be somewhat less than the total flux. Let these quantities be denoted by Φu and Φt respectively. Then the quantity known as the leakage factor is given by

$$\frac{\Phi t}{\Phi u}$$

denoting the ratio of the total flux to useful flux.

EXAMPLES

(1) Find the hysteresis loss in a mass of 100 kgs. of wrought iron undergoing 50 magnetic reversals per second if the maximum induction is 5000 C.G.S. units.

Specific gravity of iron 7·5, $\eta = \cdot 003$.

(2) Find the number of ampere turns required to produce a magnetic flux of 24,000 lines in a cast steel ring, mean diameter 20 cms., and which is made up of three equal lengths whose cross-sectional areas are 3, 4, and 6 sq. cms. respectively (Fig. 3).

(3) A wrought iron ring, 20 cms. diameter and 1 cm. diameter of cross-section is wound with 80 turns of wire. Find the current required to produce a total flux of 11,000 lines (Fig. 3).

(4) Find the ampere turns required to produce a flux of 60,000 lines in a steel ring 20 cms. diameter and 5 sq. cms. cross-section. If the ring is cut so that it has an air gap of 1 mm., find the extra ampere turns required to maintain this flux.

CHAPTER IV

INSTRUMENTS

§ 1. **Classifications.**—There are many conventions adopted for classifying measuring instruments. They are sometimes grouped under headings having regard to the purpose for which they are used, *i.e.*, amperemeters (or ammeters), voltmeters, wattmeters, etc. In nearly all direct current instruments the action depends upon the strength of magnetic field set up by the current and distinction is drawn by some writers between moving coil and moving iron types. In the former, a coil is suspended in the field of a magnet or another coil, and its motion is controlled by some force such as gravity, spring, etc., so that the deflection produced is a measure of the deflecting force.

In the latter type the coil is fixed, and a piece of soft iron is so arranged as to be able to move under some form of control. When a current flows in the coil, the iron will, of course, tend to move so as to cause as many lines of magnetic force as possible to pass through it.

Again, instruments are sometimes classified according to the method of control employed.

It should be noted that spring control is the more satisfactory method and is that which is usually adopted, the chief reasons being that, if the parts are balanced, the instrument need not be levelled, and also the deflection is proportional to the deflecting force.

For this reason, moving coil instruments are preferable to the moving iron type, since the latter are usually gravity controlled.

Another means of grouping is to distinguish between zero and deflectional or indicating types.

In the former, the moving portion must be maintained always in the same position (*i.e.*, zero position), and the couple required to maintain it there is measured as, for instance, by means of a spring and graduated torsion head.

In the latter, a pointer is attached to the moving portion and passes over a scale which gives the reading direct.

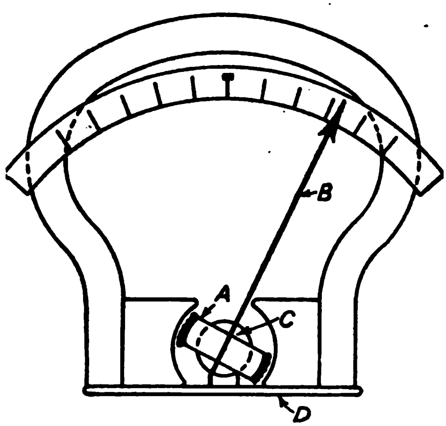

FIG. 38.—MOVING COIL INSTRUMENT

A. Moving coil on aluminium former
B. Pointer attached to A
C. Soft iron core
D. Brass strap supporting C

§ 2. **Ammeters and Voltmeters.**—When used for direct current measurement, these instruments are usually of the moving coil type.

Fig. 38 shows, diagrammatically, a Weston instrument. The control is by means of two flat spiral watch springs. These are attached one to each end of the moving coil, and form the means whereby the

current is led in and out. The permanent magnet should be long in comparison to its breadth, in order that the self demagnetizing force shall be as low as possible. The moving coil is wound on a light aluminium former, which causes the instrument to be "dead-beat," on account of the strong eddy currents induced in the former by its motion across the field.

If required to be used as an ammeter it is placed in series with the current to be measured. It follows, of course, that the resistance of the instrument should be as low as possible so that there shall be no appreciable

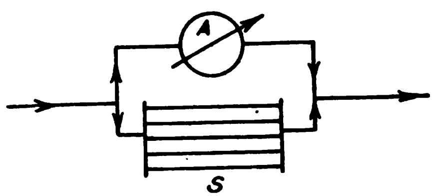

FIG. 39.—AMMETER SHUNT
A. Ammeter S. Shunt

voltage drop in it. If used as a voltmeter, it is placed as a shunt across the points where P.D. is required. From this it will be seen that the resistance must be high, so that there will be only a negligible portion of the main current passing through it. The potential drop across an ammeter is usually of the order of ·05 to ·1 volt, while the current which passes through a voltmeter varies from ·015 to ·02 amps.

§ 3. **Ammeter Shunts.**—It sometimes happens that it is required to measure high currents by means of an ammeter constructed for much lower ones. In these cases it is usual to place a low resistance shunt across the ammeter in order that only a small proportion of the main current shall pass through the instrument (Fig. 39).

Suppose the resistance of the ammeter to be R and that of the shunt S, I being the current to be measured. Then, if I_1 be the shunt current, and I_2 the instrument current, $I_1S = I_2R$ (since the voltage drops over the two must be equal).

$$\text{Hence,}\quad I_1 = I_2\frac{R}{S}$$

$$\text{and}\quad I = I_1 + I_2 = I_2\left(1 + \frac{R}{S}\right)$$

$$= I_2\left(\frac{S + R}{S}\right)$$

Thus the magnifying power of the shunt is equal to $\frac{S + R}{S}$

Suppose it is required to make this some definite value, k, then

$$k = \frac{S + R}{S}$$

$$\text{or,}\quad S = \frac{R}{k - 1}$$

Shunts are usually constructed of thin sheets of manganin or constantin soldered in parallel to end blocks of copper. These two metals possess the advantage of having a low increase of resistance with temperature. Constantin is easier to solder than manganin, but has the disadvantage of possessing a higher thermo E.M.F. in contact with copper; hence, if the two ends of the shunt are at different temperatures, a slight error may be introduced by the presence of a local E.M.F. in the shunt.

It will, of course, be obvious that a voltmeter may be used as an ammeter by shunting across it a known resistance carrying the current and dividing the reading by the resistance of the shunt.

4. Dynamometers.—In these instruments the permanent magnet is replaced by a coil conveying the same current as the moving coil, or a definite proportion of it. The deflecting couple is then proportional to the product of the currents in the two coils.

The Kelvin Ampere Balance is a "zero" instrument of this type. Fig. 40 is a picture of a complete instrument, and Fig. 41 a diagrammatic representation of it.

It consists of six coils, in series with each other, through which the current is passed. *A*, *B*, *E* and *F* are fixed, whilst *C* and *D* are fixed to a rigid arm pivoted at its centre and accurately balanced. Attached to the arm is a graduated scale on which is placed a sliding weight. It will be obvious from Fig. 41 that, when a current is flowing through the coils, *C* will be attracted to *E* and repelled from *A*, and *D* will be attracted to *B* and repelled from *F* by forces proportional to the square of the current, or, in other words, the pivoted arm is acted upon by a couple proportional to the square of the current. The sliding weight is now passed along the scale until its moment about the pivot just counterbalances this electrodynamic couple, restoring the arm to the zero position, the number of graduations being proportional to the square of the current. The scale may therefore be graduated and calibrated in terms of current in amperes. This instrument is exceedingly sensitive and accurate, and is therefore very expensive and not used for commercial purposes. It is, however, very useful for calibration of other instruments, and the instrument used by the Board of Trade for standardization purposes is of similar design.

§5. **Wattmeters.**—These instruments are seldom used in direct current work since watts are equal to the product of amps. and volts. It will, however, be

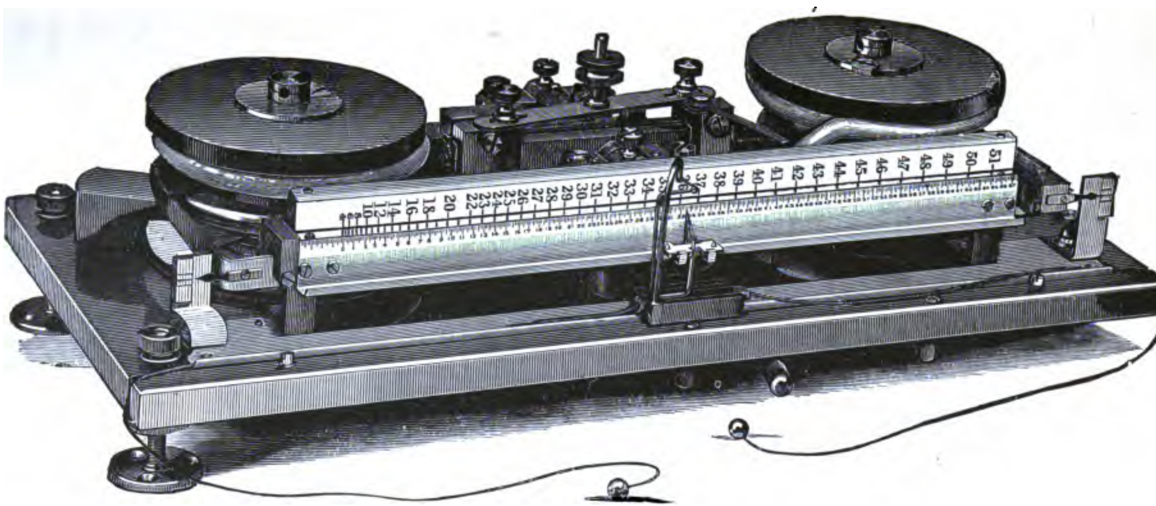

FIG. 40.—KELVIN AMPERE BALANCE

convenient to describe the method of use since they also are of the dynamometer principle. Fig. 42 shows the connections for use. *A* is a low resistance coil in

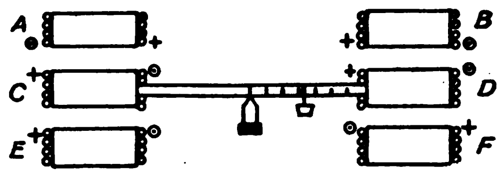

FIG. 41.—DIAGRAM OF KELVIN AMPERE BALANCE

A,B,E,F. Fixed coils
C,D. Coils attached to balance arm

series with the main current, and is known as the amps. coil. *V* is connected through a high resistance

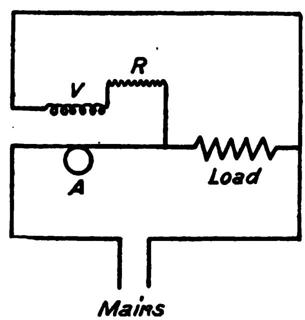

FIG. 42.—CONNECTIONS FOR WATTMETER

A. Amps coil
V. Volts coil
R. High resistance

R across the load, hence the current in *V* will be proportional to the P.D. on the load. *V* is usually the moving coil, and since the deflection produced on it is proportional to the product of the two currents,

it will therefore give a measure of watts so that the scale may be graduated accordingly.

§ **6. Recording Ammeters and Voltmeters.**—It is often required to have a record of the variation of

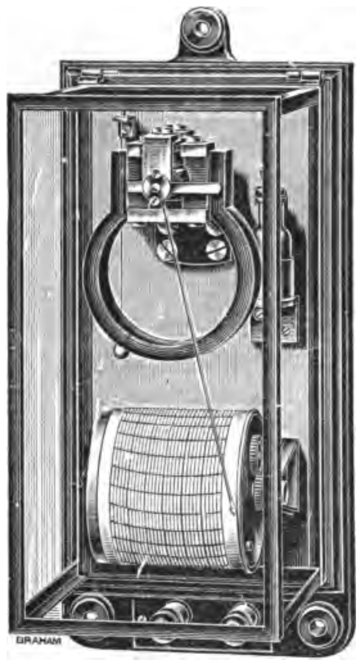

FIG. 43.—RECORDING VOLTMETER AND AMMETER

current or voltage in any particular circuit. For instance, it is a Board of Trade Regulation that, when rails are used as the return circuit in electric tramways the P.D. between the rails at any point and earth shall not exceed 7 volts. It is usual, therefore, to have a recording voltmeter connected between the rails and

earth at important junctions in order to see that this condition is fulfilled. Again, in stations supplying a fluctuating load, a recording ammeter is required so as to obtain a record of the load variation or a load-time curve, as it is called.

The electrical details of these instruments are exactly the same as in ordinary ammeters and voltmeters, but, instead of an indicating pointer, a light arm carrying a pen point is used. The pen point presses lightly on a strip of paper passing at a uniform rate round drums and graduated in amperes or volts as required. Two important points are that the moving forces must be greater if there is liable to be much pen friction, and also the damping must be made very efficient if there are likely to be great variations in current (or P.D.).

§ 7. **Supply Meters.**—It is, of course, necessary to know the total quantity of energy supplied to consumers, and supply meters are used for this purpose. They measure the value of $\int E \,.\, I \,.\, dt$ (*i.e.*, total watt-hours supplied). As, however, E, the voltage, is usually fairly constant, it is more frequent, especially with small consumers, to use an instrument which measures the quantity of electricity used (*i.e.*, $\int I.dt$), which, multiplied by the voltage, gives the total watt-hours. These instruments are not so accurate as the others, taking, as they do, no account of variation in P.D. of supply, but they possess the advantage of being considerably cheaper.

The former are called watt-hour meters, and the latter quantity meters or integrating ammeters.

Supply meters may be divided into three main classes: motor-meters, clock-meters, and electrolytic-meters. The last are essentially "quantity" meters,

since they measure the amount of electrolysis which has taken place in the time (*i.e.*, the amount of electricity which has passed).

§ 8. **The Elihu Thomson Meter.**—This is an energy meter of the motor type. It is shown diagrammatically in Fig. 44. An armature, with its commutator connected by means of brushes through a high resistance

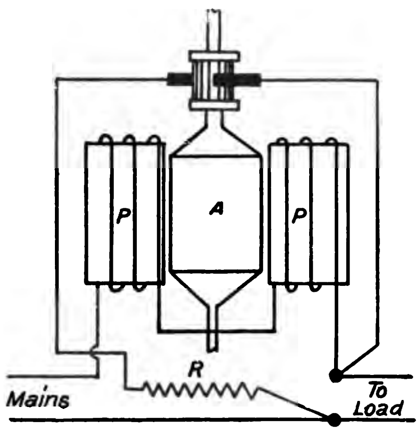

FIG. 44.—ELIHU THOMPSON MOTOR METER

A. Rotating armature
PP. Poles
R. High resistance in series with A

across the P.D. of supply, rotates in finely machined bearings between two pole pieces, round which are coils carrying the main load current. The axis of the armature is connected through worm gearing to the counting train. At the lower end of the armature axis is fixed a copper disc (not shown) rotating between permanent magnets. This, therefore, acts as an eddy current brake giving a retarding torque proportional to the speed. The armature is of non-magnetic material so that the field is weak and on the straight

portion of the $B\,H$ curve. It is therefore proportional to the current. Hence, the driving torque is proportional to $E\,I$ and, since the retarding torque is proportional to the speed, the number of revolutions made is proportional to the number of watt-hours supplied. In order to compensate for friction, which

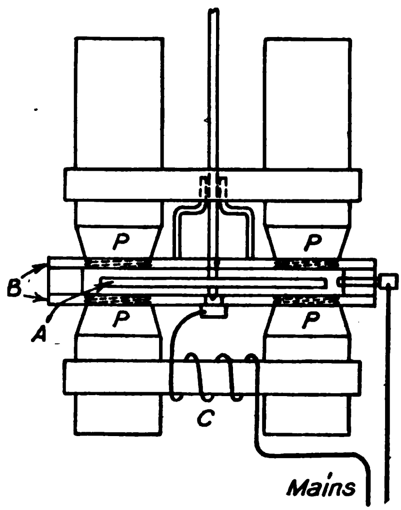

FIG. 45.—FERRANTI METER

P. Pole pieces
A. Copper disc
B. Brass plates
C. Friction compensating coil

is assumed to be constant, a compensating coil (not shown) is wound on one of the pole pieces inside the main field coil. This is in series with the armature, and therefore carries a constant current. It may thus be arranged to give a torque equal to that due to friction. For 100 volt instruments, the resistance of the armature circuit is about 300 ohms, more for larger sizes and less for smaller.

§ 9. **Ferranti Meter.**—This is a motor meter of the "quantity" type. A copper disc, whose axis is connected through worm gearing to the counting train, rotates in a mercury bath between the poles of two permanent magnets placed side by side. (See Fig. 45.) The mercury is contained between two brass plates, separated by a fibre ring and prevented

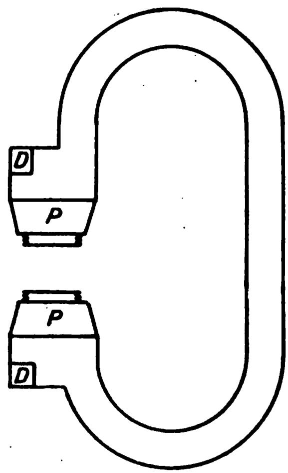

FIG. 45 (*a*).—SIDE VIEW OF MAGNET OF FERRANTI METER
PP. Poles
DD. Soft iron cross pieces

from contact with the mercury by discs of presspahn. Current passes into the mercury by means of a metal stud passing through the fibre ring. The greater portion passes radially through the disc to the centre and is led out through the vertical axis. The magnet on the right supplies the driving torque, and braking is provided by means of eddy currents induced in the disc by the magnet on the left.

Since the braking torque is proportional to the

speed, the number of revolutions will be a register of the number of ampere hours. Two iron bars are fixed across the magnets as shown, and the current is led through a few turns of wire round the lower bar. This causes a strengthening of the right-hand poles and a weakening of the left-hand ones, and thus, by increasing the driving torque and reducing the braking, compensates for mercury friction.

§ **10. The Aron Clock-meter.**—In this instrument, two exactly similar pendulums are placed side by side. Immediately below them are placed two coils or solenoids carrying the main load current. The pendulum bobs also carry solenoid coils connected through a high resistance across the mains. When currents are flowing their directions are such that one pendulum is attracted and the other repelled. Thus the one will be accelerated and the other retarded. But the forces acting on the pendulums are proportional to the product of the currents in each pair of coils (*i.e.*, to the power). It may be shown mathematically that, within limits, the increase and decrease in the number of oscillations per second of the pendulums are proportional to the forces of attraction and repulsion respectively. Hence, it follows that the total energy supplied in a given time is proportional to the difference between the vibrations made by the two pendulums in that time.

The pendulums are driven by a main spring through an ordinary clock escapement, the mainspring consisting of a short length of flat strip steel which is wound up automatically by an electro-magnet every half minute.

The winding apparatus consists of a two-pole electromagnet (Fig. 46) with a laminated Z-shaped core arranged as shown and pivoted at its centre. When the magnet is excited it draws the core round in a

clockwise direction, thus winding up the spring. It is connected through a ratchet wheel and train to the escapement and thus drives the pendulums. Arrangement is made so that when the spring is fully wound a contact is broken and the exciting current ceases to flow. As the pendulums oscillate the core will slowly return to the position shown in Fig. 46, when contact is again made and the process repeated. This takes place once every 30 seconds.

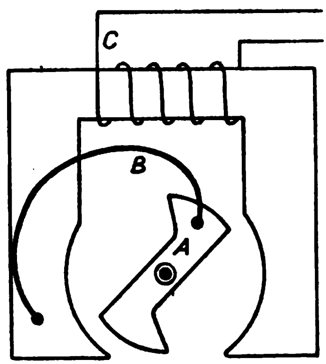

FIG. 46.—WINDING APPARATUS OF ARON CLOCK METER

A. Pivoted iron core
B. Spring retaining core in position shown
C. Exciting coil

The general arrangement of electrical connections is shown in Fig. 47.

The means by which drive is given to the counting train is shown in Fig. 48.

The pendulums actuate toothed wheels T and T' in opposite directions, and these each engage with the planet wheel P. So long as the speeds of T and T' are equal and opposite the wheel P will simply rotate on its axis without moving round the shaft A. If, however, the speeds are different, P, in addition

to rotating on its axis will revolve round A at a speed equal to half the difference between the speeds of T and T'. The wheel P is balanced by a counterpoise W.

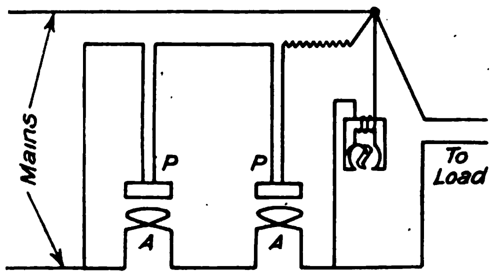

FIG. 47.—CONNECTIONS OF ARON CLOCK METER

P. Pendulums
A. Load current coils

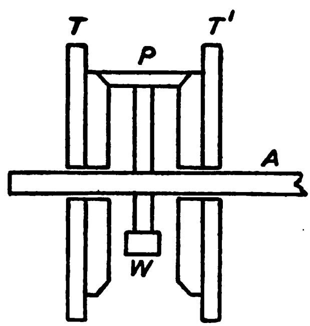

FIG. 48.—DIFFERENTIAL DRIVE OF ARON CLOCK METER

TT[1]. Toothed wheels
P. Planet wheel
W. Counter poise to P
A. Driving shaft to counting train

Rotation of A is transmitted through gearing to the train of counting wheels, which can thus be graduated in watt hours.

It is difficult, and therefore expensive, to make the natural periods of the pendulums exactly equal, and so the following device is adopted. They are made as nearly as possible equal by means of adjustable balance nuts, and the errors are eliminated by causing a reversal of currents through the pendulums every 10 mins. This will cause the attracted pendulum to become the repelled one and vice versa.

Since this causes a reversal of rotation of A, means are introduced whereby the direction in which the counting train is driven by A is reversed at the same moment. Hence, errors are eliminated and the counting train is driven in a constant direction.

§ **11. The Wright Electrolytic Meter.**—As already stated, electrolytic meters depend upon some form or other of electrolytic decomposition. The amount depends upon the number of ampere hours, and hence they may be classified as quantity meters. Some depend upon the deposition of copper or silver, but possess the disadvantage that they must be weighed to find the number of ampere hours. Others are arranged to give a reading direct, and perhaps one of the best of that type is that originally devised by A. Wright and improved by Hatfield. In the main it consists of a sealed glass tube containing a saturated solution of a double nitrate of potassium and mercury. Current is passed into this through an anode of pure mercury and out by a cathode of iridium. These are arranged at the top of the tube, as shown in Fig. 49. The mercury is held in place by a double ring of glass rods.

Electrolytic action causes drops of mercury to form on the iridium plate, and these fall to the bottom of the tube by gravity.

A scale is arranged alongside the tube, and the height of the mercury indicates the number of Board of Trade units consumed.

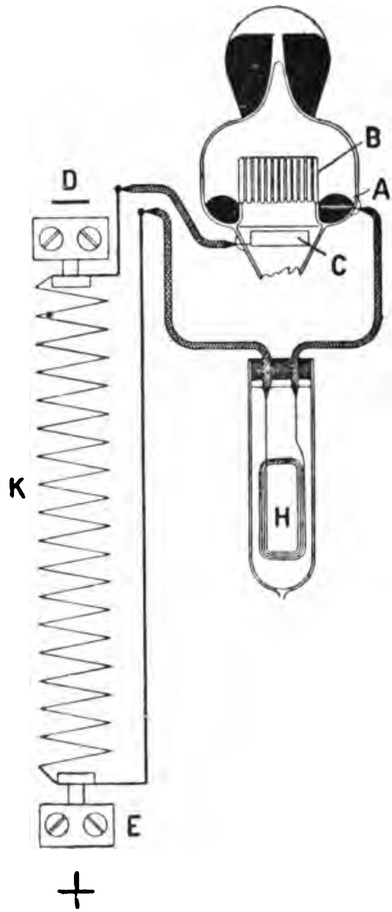

FIG. 49.—DIAGRAM OF ELECTROLYTIC METER

A. Mercury
B. Glass fence
C. Iridium cathode
K. Shunt
H. Series resistance

In older types of instruments a cone of gauze was used instead of the double ring of glass rods, but this became choked with mercury after a time.

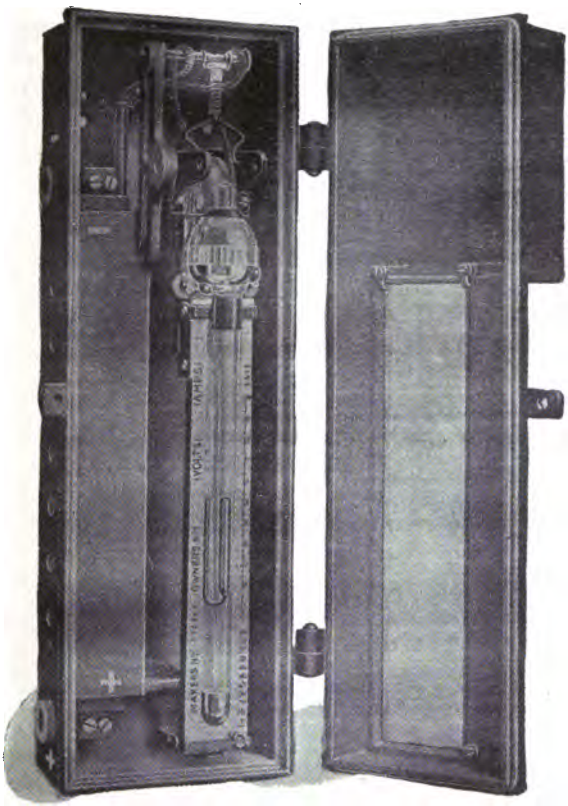

Fig. 50.—Electrolytic Meter

Mercury supply is maintained by means of a reservoir at the side of the tube, and, in the larger types of instruments, the tube is in the form of a syphon, so

that the mercury returns to the reservoir after 100 units have been consumed.

As with nearly all electrolytic meters, a shunt is employed, and this must have about the same temperature coefficient as the meter; also, since the internal resistance of the meter is low, a resistance must be placed in series with it to make the shunt effective. The general view is given in Fig. 50.

These instruments are accurate even at small loads, and there is small chance of a breakdown. They require, however, a long time for calibration (which is only necessary once, as a rule), and will fail to record if connected the wrong way.

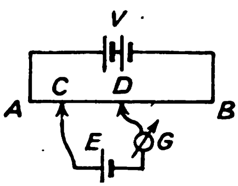

FIG. 51. — PRINCIPLE OF POTENTIOMETER

§ 12. Crompton Potentiometer. —An important adjunct to the testing of ammeters and voltmeters is the potentiometer. Its main principles are as follows. If a source of potential V (Fig. 51) be connected across the ends of a uniform resistance AB, the fall of potential down AB will be uniform. If an unknown source of potential E be connected through a galvanometer G to variable points CD, such that no deflection is produced on the galvanometer, we know that the P.D. of E is equal to the potential drop down DC. We therefore have a means of determining E from the ratio of DC to AB.

The resistance of the Crompton Potentiometer consists of 14 resistance coils P (Fig. 52) and a graduated uniform wire W.

In addition there are resistance coils R for rough adjustment and a continuously variable resistance R_1. Contact is made with the graduated wire W by means of a slider S in contact with it, and a "contact

wire," W' stretched parallel to it. W' is of negligible resistance.

The coils P are each of 10 ohms resistance, and the slide wire is 10 ohms divided into 100 equal parts.

To use the instrument there is included at V a secondary cell which gives a fairly constant current

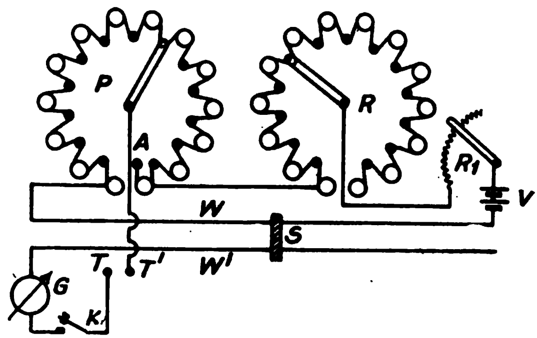

FIG. 52.—DIAGRAM OF CROMPTON POTENTIOMETER

P. Resistance coils
W. Slide wire
W^1. Contact wire
RR^1. Adjusting resistances
G. Galvanometer

for an appreciable length of time. The arm of P is moved to the point A and the slider placed at division 34 on W. TT' then includes 143·4 ohms. A Clark standard cell (whose E.M.F. at 15° C. is known to be 1·434 volt) is connected across TT' and then, leaving the slider and P unaltered, R and R' are adjusted until there is no deflection on the galvanometer when the key K is tapped.

We then know that every 100 ohms of P and W represent a drop of 1 volt.

If the Clark cell be now removed and an unknown P.D. substituted, R and R' are left and the slider and P adjusted till, once more, no deflection is obtained on the galvanometer. If, for example, there are 8

coils of P and 56 divisions of the slide wire, we know the resistance is 85·6 ohms which represents ·856 volt.

If the voltage to be tested is greater than about 1·5 volts, means must be adopted to reduce it to this amount, and for this purpose a "volt-box" is used (Fig. 53).

The unknown P.D. is connected across a uniformly varying resistance XY, and tappings are made from X and Z to the terminals TT' such that the ratio of XZ to XY is sufficient to reduce the voltage to below about 1·5 volts. The result obtained must then be multiplied up in the ratio XY to XZ.

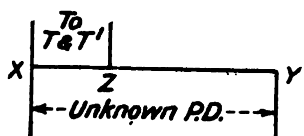

FIG. 53.—DIAGRAM OF "VOLT-BOX"

§ 13. Calibration of Ammeters and Voltmeters.—If standard instruments whose accuracy is known are available, a simple method, of course, consists in comparing the instruments under test with the standards.

Ammeters must be connected in series and voltmeters in parallel with each other.

A very accurate means of calibrating an ammeter consists in placing the instrument in series with either a copper, or, preferably, a silver voltmeter.

The electrolyte of the voltmeter consists, in the first case, of copper sulphate, sulphuric acid and water, and, in the second case, of silver and potassium cyanides. It is known that ·330 milligrammes of copper or 1·130 milligrammes of silver are deposited on the cathode by the passage of 1 coulomb. If, therefore, a constant current be kept flowing for a definite period, and the cathode be carefully weighed before and after, a very accurate measure of the current may be obtained.

The current densities should be between ·065 and ·1 amp. per square inch of cathode for copper, and between ·015 and ·030 per square inch of cathode for silver. Although the weights given above per coulomb are those which are actually liberated, the amounts which adhere to the plates (*i.e.*, the amounts determined by weighing) vary slightly for different current densities, and tables are prepared giving the "apparent" electric chemical equivalent weights, as they are called. The actual variation between the limits of current density given above is not more than about 0·2 per cent.

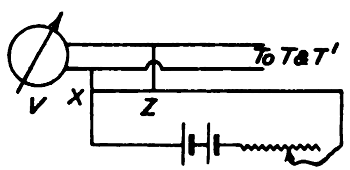

FIG. 54.—CALIBRATION OF VOLTMETER BY POTENTIOMETER

The above is a very good means of calibrating an ammeter which obeys a definite law. If it does not do so, however, the method is inconvenient, since a calibration must be made for every point on the scale.

A voltmeter is best calibrated by the potentiometer method. The potentiometer is first adjusted by means of a standard cell, as explained in par. 12, so that each division on the slide wire represents ·001 volt.

A battery of secondary cells is then connected across a resistance from which different tappings (*i.e.*, different P.D.'s) may be obtained within the range of the voltmeter under test. Its reading is then compared with that obtained when the same points are connected to the terminals of the potentiometer.

If the voltmeter to be tested is not below about

1·5 volts, use is made of a " volt-box " of the required reduction ratio. An ammeter may be calibrated by means of a potentiometer if a standard resistance is available. The P.D. between the ends of the resistance is then measured and the current calculated by dividing it by the resistance.

Similar methods by which supply meters may be tested with standard ammeters and voltmeters will, of course, be obvious to the student.

EXAMPLES

(1) It is required to measure currents between 10 and 15 amps., and the only instrument available is an ammeter reading to 1·5 amps. What would you do? Give calculations.

(2) The values of currents up to 20 amps. are required to be known, and the only instrument available is a millivoltmeter graduated from zero to ·1 volt.

What standard resistance would you require, and how would you use it?

(3) In a certain voltmeter reading to 50 volts, the resistance of the moving coil is ·6 ohm, and the added resistance is 999·4 ohms. If this added resistance can be removed, how would you arrange the instrument as an ammeter reading up to 10 amps.? If you use a shunt, what would be its value?

CHAPTER V

STORAGE BATTERIES

§ 1. **Advantages.**—Storage batteries, secondary batteries, or accumulators, as they are variously termed, form, as their name implies, a means whereby electric energy may be stored and retained until required for use. Their advantages and uses are many, but the following may be cited as the most important.

(1) In the event of a temporary breakdown of one of the generators at a power station, its load may be supplied by a battery of cells while the necessary repairs are carried out, thus avoiding overloading the remaining machines.

(2) During the time of "peak" load on a power station (*i.e.*, a load more than usually high) the excess load may be taken on a battery, so that there is then no necessity for the installation of unduly large machinery.

(3) At times when the load is small, it might be possible to shut down altogether and work on the battery only, thus saving running expenses.

It is well known that the most economical running of a plant would be obtained if the load could be kept at a constant value. It will be realized, therefore, that it should be possible to arrange a suitable combination of generators and battery, so that the latter supplies the excess current during periods of heavy load, and is charged during periods of light load. This would tend to produce the desired effect.

§ 2. **Composition of a Cell.**—A storage cell is simply a practical application of the ability to produce chemical reactions by means of an electric current (*i.e.*, electrolysis).

In the past many substances have been tried for the plates. Copper, cadium, zinc, cobalt, etc., have all had their turn, but all possess great disadvantages which are not present, or only slightly so, in lead plates. The electrolyte used is dilute sulphuric acid, although alkaline and neutral solutions have been tried without, however, any success. The only other combination which at present seems to stand any chance of competing with the lead-sulphuric acid cell is the iron and nickel cell of Edison. The electrolyte used with this is a concentrated solution of caustic potash.

It may be said here that the advantages of this latter lie in traction work rather than the heavy work demanded from a modern power plant.

Planté devised the original plates in or about 1860, and did much work on the subject for the next 30 years. He used two ordinary sheets of lead, insulated from each other by a sheet of indiarubber, and rolled them up together, immersing them in dilute sulphuric acid. He then submitted them to a series of cyclic charges and discharges until the chemical reactions had eaten well into the plates. There gradually came to be adopted such devices as corrugations, etc., on the plates, in order to present a larger surface to the electrolyte.

Plates constructed as above are known alternatively as Planté type or " formed " plates.

Later on, Faure suggested and elaborated a scheme whereby the plates are made in grid form of lead, and a paste of red lead (Pb_3O_4), or litharge (PbO), mixed with dilute sulphuric acid, is pressed into the cavities. These plates take less time to bring into the required chemical and physical condition than the formed plates, but are more liable to disintegration under heavy discharges, which tendency is, however, more

pronounced in the positive than the negative plates. In cells, therefore, which are liable to receive large demands for current, such as those used in traction work it is customary to use Planté or formed plates for the positive, and Faure or pasted plates for the negative.

§ 3. Chemical Formulae.—No absolute chemical formula can be given since the correct constitutions of the plates are by no means certain.

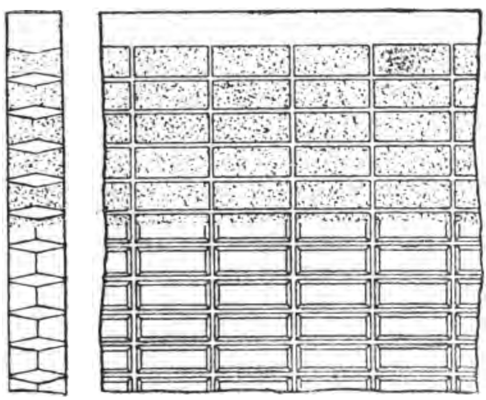

FIG. 55.—PASTED POSITIVE—RECTANGULAR GRID

During charge, however, the sulphuric acid is split up into ions of H_2 and SO_4, and these combine with the negative and positive plates respectively. We may represent this by

$$-^{ve} \text{ plate}, \quad H_2 + PbSO_4 = Pb + H_2SO_4$$

$$+^{ve} \text{ plate}, \quad SO_4 + PbSO_4 + 2H_2O = PbO_2 + 2H_2SO_4$$

We see, then, that the lead sulphate which, before charging, covered both plates, has now become plain "spongy" lead on the negative plate and lead peroxide on the positive.

In discharging, we have

$-^{ve}$ plate, $SO_4 + Pb = PbSO_4$

$+^{ve}$ plate, $H_2 + PbO_2 + H_2SO_4 = PbSO_4 + 2H_2O$

As regards general appearance, in the discharged cell the positive plates are reddish brown and the negative a greyish white. In the charged cell the positive plates are a deep chocolate brown, and the negative plates a deeper grey.

It must be pointed out that the actual lead sulphate in the cells is not $PbSO_4$, for this is insoluble. It probably contains a higher proportion of lead.

When charging is complete, or nearly complete, there is little lead sulphate left, and the H_2 is left alone and rises to the surface in the form of bubbles. This is known as "gassing," and when the cells gas freely it is an indication that the charge is nearing completion.

In the Edison cell the reactions are even less certain than with the lead cell. A near approximation is probably the following, where the left-hand side represents the charged condition and the right-hand side the discharged—

$$NiO_2 + Fe \rightleftarrows NiO + FeO$$

It should be noted that the electrolyte caustic potash does not appear at all, and, in practice, the density is found to vary almost imperceptibly between charge and discharge.

The E.M.F. developed by an Edison cell is about 1·5 volts immediately after charge, and the mean voltage of discharge about 1·1 volts. Its chief advantages are lightness and strength under overload. It can also be left uncharged for longer than the lead cell without deterioration.

§ **4. Charge and Discharge Curves.**—The E.M.F. of a charged lead cell is about 2 volts, but varies slightly with the specific gravity of the electrolyte. Normal

limits for the specific gravity are about 1·21 and 1·18 for a charged and discharged cell respectively.

Fig. 56 shows the charge and discharge curves. Discharge must not be carried below 1·8 volts, otherwise the E.M.F. will fall rapidly, as shown by the dotted portion of the discharge curve, and will damage the cell, lowering the capacity and causing insoluble

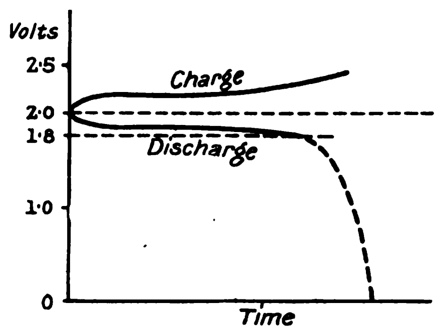

FIG. 56.—CHARGE AND DISCHARGE OF SECONDARY CELL

white sulphate to form. It is difficult to get rid of this once it has formed, chiefly owing to its insolubility and high electrical resistance.

It will be noted that the charge curve rises to as much as 2·5 volts, though if the charged cells be left to themselves the voltage will fall gradually to its normal value.

The reason is probably the fact that the specific gravity of the acid, which has been rising during charge, as shown by the equations in par. 3, is greater in the immediate vicinity and in the interstices of the plates than elsewhere, thus giving the higher E.M.F. When the cell is left, however, the acid gradually diffuses and assumes a mean value somewhat lower.

Stronger solutions of sulphuric acid are not used because it then becomes liable to attack the plate. Furthermore, after a certain strength (specific gravity of about 1·22), the specific resistance of the acid rises, thus increasing the internal resistance of the cell.

§ 5. Methods of Charging.—There are two methods of charging, viz., at constant P.D. and at constant current, but the former is practically confined to the Continent.

For charging, either a separately excited or else a shunt wound generator must be used, but on no account a series wound machine. The reason for this will be seen in Chapter X. The positive pole of the generator is connected to the positive end of the battery, and negative to negative. Now the P.D. which is applied to the battery has to overcome the total back E.M.F., and also the voltage drop due to the current passing through the internal resistance of the cells. It will be seen therefore, that, since the E.M.F. gradually rises, the P.D. must be correspondingly increased if the current is to be maintained constant. This may perhaps be best illustrated by a simple example.

A battery of 50 cells having been discharged to an E.M.F. of 1·8 volt each, is to be charged at a constant current of 40 amps. The internal resistance of each cell is ·002 ohm. Find initial and final charging P.D. if the E.M.F. at the end of charge is 2·5 volts each cell.

At commencement of charge, the back E.M.F. $= 50 \times 1{\cdot}8 = 90$ volts, and the resistance drop $= 50 \times {\cdot}002 \times 40 = 4$ volts. Hence, initial P.D. $= 90 + 4 = 94$ volts.

At the end of charge the back E.M.F. $= 50 \times 2{\cdot}5 = 125$ volts, and the resistance drop as before $= 4$ volts. Hence, the final P.D. $= 125 + 4 = 129$ volts.

The amount of current required will depend upon the

number of plates in parallel, and is usually stated by the makers. It should not exceed 7·5 amps. per square foot of positive plates in parallel. This is not the total area of surface on both sides, but only of the outline of the plate. Thus, if all the cells are in series, and each cell contains six positive plates, each ·75 sq. ft. area, the charging current must not exceed 6 × ·75 × 7·5 amps. = 34 amps. (approx.).

If the generator which is charging the battery is not supplying any other circuit with power, its voltage may be gradually raised as the charge proceeds by means of a rheostat in the field circuit. If, however, other circuits are being supplied by the main generator, an auxiliary generator, called a booster, is used in series with the main generator, so as to avoid altering the main P.D. (*See* Fig. 57).

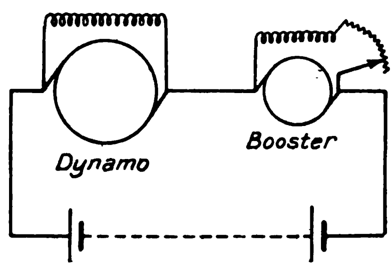

FIG. 57.—CHARGING BATTERY USING BOOSTER

An electro-magnetic break is arranged in the cell circuit, so that, if for any reason the generator should fail, the circuit will be broken and the cells will not then be able to discharge through the inactive dynamo.

Small cells may be charged from electric light mains, (so long as they are direct current). It was formerly customary to place the cell across the mains in series with a lamp. This, however, is not always possible now, since modern lamps are so efficient and take only small currents. (A 200 candle power lamp on 200 volt mains will only take about ·5 to ·75 amps.)

A variable resistance and ammeter are therefore required instead of the lamp.

6. Efficiency.—The three main points to be considered with regard to a storage battery are—

1. Capacity.
2. Efficiency.
3. Internal resistance.

(1) This does *not* mean, as the name appears to imply, the amount of electricity which the battery is capable of receiving, but the amount which it is

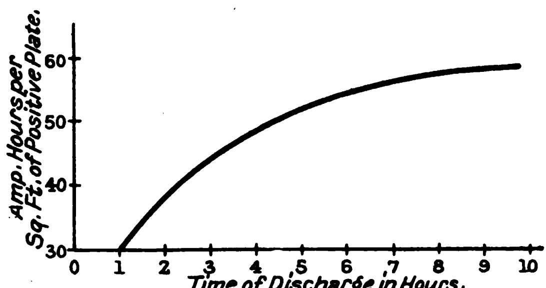

Fig. 58.—Variation in Capacity with Rate of Discharge

capable of delivering after being fully charged. It is usually reckoned in ampere hours. Thus, a battery with a capacity of 1000 ampere hours can deliver a current of 100 amperes for 10 hours, or 200 amperes for 5 hours, etc. The capacity is not, however, constant at all discharges but decreases at higher loads.

At lower loads than a 10 hour rate the capacity is fairly constant, but above that it decreases with the load. The curve in Fig. 58 gives the approximate variation in capacity of the Chloride Company's batteries in ampere hours per square foot (not surface) of positive plate for various times of discharge.

A rough rule for ascertaining the capacity of a battery is to take 60 to 80 ampere hours per square foot of positive plate at the low discharge rate,

It should be stated that at the 1 hour discharge rate the E.M.F. may be allowed to fall to 1·65 volts per cell.

(2) There are two efficiencies to a battery, (*a*) the ampere hour efficiency and (*b*) the watt hour efficiency.

(*a*) is the ratio of ampere hours of discharge to ampere hour charge, and depends upon the condition of the cell and the rates of charge and discharge. It is usually between 80 and 90 per cent.

(*b*) is the ratio of watt hours of discharge to watt hours of charge, and is somewhat lower, between about 70 per cent. and 80 per cent. In general, it may be stated that efficiency increases with decreasing rate of discharge.

(3) The internal resistance is usually very small in good accumulators, and the greater portion is due to the contact of the plates with the electrolyte. Batteries are therefore constructed with several positive and several negative plates in parallel in each cell, arranged alternately only a small distance apart, separated by distance pieces of an insulating material.

Fig. 59 shows a cell with the plates built up and insulated from each other by glass rods, which may be seen projecting below them. Sometimes plates of some porous material are used instead of the glass rods. There should always be one more negative plate than positive, so that each of the latter has a negative on either side of it.

If this were not done the outside positive would expand unequally on the two sides and buckle. Negative plates are not affected like this.

As a rough rule, the internal resistance of a cell may be taken to be 1·5 ohms per square inch of total surface of positive plates. The greater the surface the less the resistance. It varies from ·1 ohm in a small cell to ·001 ohm in a large one.

§ 7. **Tests.**—To test capacity and efficiency the

battery must always be reduced to what is known as a cyclic state. That is, it must be given a succession of charges and discharges at a constant rate before making the test. That is in order that the active material may be in its normal state.

It is analogous to the necessity for keeping iron on the cyclic curve when making *BH* tests.

All that is then necessary is to have a recording

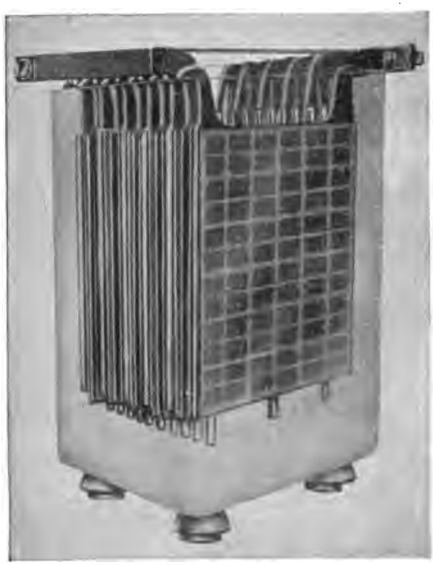

FIG. 59.—SECONDARY CELL

ammeter and a watt-hour meter, so that variations of current are marked on a chart against a time base, and the values of ampere hours and watt hours are obtained for charge and discharge, and the ratio taken.

To measure the internal resistance of a cell it is not sufficient merely to take voltmeter readings on open

and closed circuits and divide the difference by the current flowing, although, theoretically, this should give the required amount. The method usually adopted is that shown in Fig. 60.

Two cells B are connected to the ends of a resistance XZ. The cell under test is connected through a resistance R to an ammeter A. The positive terminal of E is connected to the positive end X of the resistance XZ. A millivoltmeter V has one terminal connected to the negative pole of E, and the other to a loose wire with which contact may be made with XZ. R is adjusted so that a suitable current flows through A,

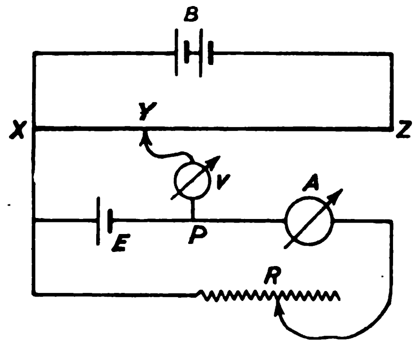

FIG. 60.—MEASUREMENT OF INTERNAL RESISTANCE

XZ. Uniform resistance
E. All under test
A. Ammeter
V. Millivoltmeter
R. Variable resistance

and then the loose wire of V is touched at various points on XZ until a point Y is found, such that no deflection is produced on V. The points P and Y must then be at the same potential, hence the fall of potential down XY is a measure of the P.D. of the terminals of E with a current A flowing through it. If now contact is broken at R, there will be a deflection on V. There is no current flowing through E, consequently the fall of potential down to XEP must equal the E.M.F. of the cell. But we saw that the fall of potential down XY was equal to the P.D. of the cell when a current A was flowing; consequently,

the reading on V must equal the difference between E.M.F. and P.D.

If, therefore, we divide this reading by the current A we shall obtain the value of the internal resistance.

§ 8. **Care of Battery.**—The principal points which need attention in the care of a battery are as follows—

(1) Never discharge below 1·8 volts P.D. per cell with current flowing. (An exception is made, as already stated, in the 1 hour rate when the P.D. may fall to 1·65 volts.)

(2) Do not leave the battery too long after discharge before re-charging, or else the plates will become sulphated.

(3) If necessary to leave it, charge fully until all cells gas freely and then re-charge about once every fortnight.

(4) If the period of inactivity is likely to extend over several months, discharge to 1·8 volts, remove the acid and fill with distilled water, then continue discharge as low as possible, finally short circuiting the plates.

Remove the water and thoroughly dry the plates. An alternative method is to charge the battery fully and then remove the acid and dry the plates.

(5) The working rates of charge and discharge should not be exceeded, or else the capacity and life of the battery will suffer.

(6) The level of the electrolyte should be $\frac{1}{2}$ in. above the top of the plates.

(7) To counteract evaporation it is necessary to raise the level of the electrolyte from time to time. This should be done by the addition of pure distilled water only.

(8) The plates should not rest on the bottom of the containing vessel but should be supported. If any active material becomes loosened by, say, an over load,

FIG. 61.—BATTERY OF LARGE SECONDARY CELLS

it will fall to the bottom and not short circuit the plates.

(9) A call may occasionally become weaker than the rest of the battery. It will require to be cut out and given a charge by itself. Separate cells are sometimes kept for this purpose and are called " milking " cells.

(10) When mixing acid, which requires skilled attention, acid should be added to water and not vice versa. The mixture becomes hot and should be allowed to cool before taking the specific gravity.

EXAMPLES

(1) The total area of surface of positive plates per cell in a battery of 20 cells is 5·8 sq. ft. Find the approximate internal resistance of the battery with all cells in series. There are three positive plates per cell. What is the maximum permissible value of charging current ?

Assuming the battery to be charged at a constant current of this value, what will be the initial and final applied P.D. if the cells were discharged to 1·8 volts each and are charged to 2·5 volts each ?

(2) The above battery was reduced to a cyclic state by a succession of charges and discharges at the constant current determined above. If the table below gives the values of P.D. at various times during charge and discharge, find (*a*) the ampere hour efficiency, (*b*) the watt-hour efficiency, (*c*) the capacity of the battery.

Time in hours.	0	·2	·3	·5	1·0	2·0	3·0	4·0	5·0	6·0	6·5	7·0	7·5	8·0	8·5
Charge P.D. .	38	42·8	42·9	43·2	43·2	43·4	43·8	44·2	44·8	46·0	47·2	49·0	51·0	52·0	52·4
Discharge P.D.	40·2	39·6	39·6	39·6	39·6	39·4	39·2	39·0	38·5	38·0	37·6	37·0	36·0	–	–

CHAPTER VI

THE DYNAMO

§ 1. Brief History.—Use was first made of the discovery that an E.M.F. is induced when a conductor is moved across a magnetic field in 1831, when Faraday constructed the first magneto electric machine or, as we now term it, dynamo.

One of his first machines of that type was as shown in Fig. 62.

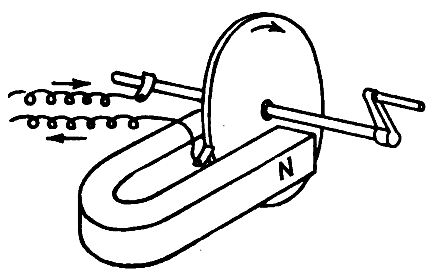

FIG. 62.—ELEMENTARY MAGNETO ELECTRIC MACHINE

A copper disc was rotated between the poles of a magnet and rubbing contacts were made at the shaft and rim. With polarity and direction of rotation as shown, the current would flow from shaft to rim.

Later on, bobbins of insulated wire were used, but, as the currents induced alternated in direction, a commutator was fixed so that the currents in the external circuit should all be in the same direction. This is shown diagrammatically in Fig. 63, which shows the form of split copper commutator adopted by Sturgeon in 1836.

With the coil rotating so that the upper portion is

moving towards the observer, the currents would be as indicated by the arrows.

§ 2. **Dynamo-electric Machines.**—In 1867, after suggestions by various people, a machine was constructed in which the current generated was used to excite the field magnets (as they are termed).

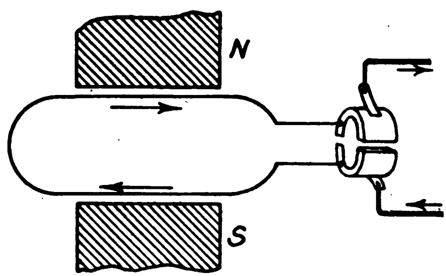

FIG. 63.—PRINCIPLE OF COMMUTATOR

The name dynamo-electric machines is given to such to distinguish them from the permanent magnet generators.

Simple machines of the above descriptions are, of course, only capable of a very small output, and then at a very low efficiency, and during the last 50 years tremendous strides have been made in dynamo design, until to-day we have direct current machines capable of very high outputs and efficiencies of the order of 95 per cent.

§ 3. **Necessity for Method of Connection of Armature Conductors.**—We have seen that the E.M.F. produced in a wire of length l, moving with velocity v across a field of intensity B, is $Blv \times 10^{-8}$ volts. Now, since B rarely exceeds 10,000 and peripheral velocity 5,000 ft. per minute, or 2,540 cms. per sec., we see that the E.M.F. in each wire will not exceed ·254 volt per cm. length, or 7·74 volts per foot length.

Some method of winding must, therefore, be adopted by which many of the conductors may be connected in series so as to add the E.M.F.'s in order to produce anything of practical utility.

§ **4. Simple Ring Winding.**—Let us consider a simple two pole ring winding as shown in Fig. 64.

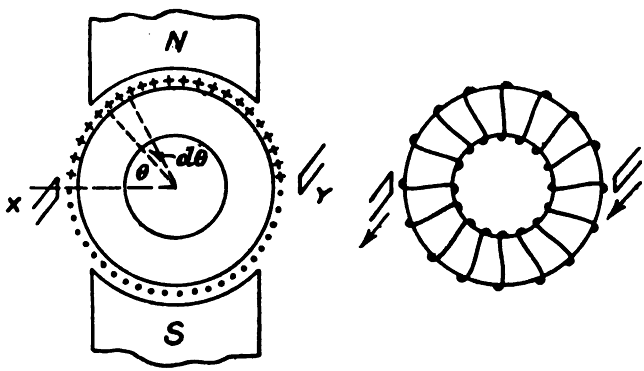

FIG. 64.—SIMPLE RING WOUND TWO-POLE DYNAMO

The armature core is composed of iron, hence, no flux passes through the air in the centre of the ring, and thus no lines are cut by those portions of the winding lying inside the ring.

It is easily seen by applying Fleming's right-hand rule that, with a clockwise rotation of the armature, the direction of the E.M.F.'s will be as shown, those under the north pole being down into the paper and those under the south pole being up out of the paper.

Let v = linear velocity of wire
l = length of conductor cutting flux
B = air gap induction
σ = wires per cm. of periphery
r' = external radius of armature
Φ = polar flux

We have E.M.F. in wire at angle θ

$$= Blv \sin \theta$$

$\therefore$ E.M.F. in wires in arc $d\theta$

$$= Blv \sin \theta \times \sigma r d\theta$$

since there are $\sigma r d\theta$ wires in series in the arc $d\theta$.

Hence, total E.M.F. from X to Y

$$= \int_0^\pi Blv\sigma r \sin \theta d\theta$$

$$= 2Blv\sigma r$$

But $Bl \times 2r = \Phi$

Hence, E.M.F. $= \Phi v \sigma$

Now if n = revs. per sec. and N = No. of wires.

$$v = 2\pi r n$$

$$\therefore \text{E.M.F.} = E = \Phi \times 2\pi r n \times \frac{N}{2\pi r} \times 10^{-8} \text{ volts}$$

$$= \Phi N n \times 10^{-8} \text{ volts.}$$

§ **5. Forms of Windings.**—The method of winding described above is, in practice, little used for a variety of reasons. Chief are the fact that the process of winding is laborious, and also that there is a great amount of inactive (*i.e.*, non-flux cutting) copper in those portions lying inside the ring. The two arrangements chiefly employed are Lap and Wave. There are various other more or less complex windings which are, however, beyond the scope of this book.

The actual conductors are of two types: "bar" and "coil."

The "bar" winding, as its name implies, consists of bars of copper embedded in slots in the armature (the reason for the slots will be seen later) and the method of connecting the ends being determined by the type of winding (Lap or Wave) which is being adopted.

The "coil" winding consists of coils of several

turns of cotton covered copper wire wound on a suitably shaped former, bound together with cotton tape and embedded in the slots as before, with the ends of the coil projecting.

Since there are several lengths of wire conductor in each slot, this method gives a higher E.M.F. than could be obtained with one bar conductor. Small dynamos, in which space is a consideration, are accordingly wound thus as a general rule.

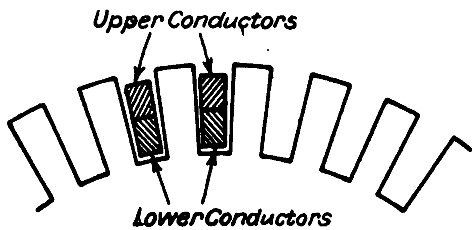

FIG. 65.—END VIEW OF "BAR" CONDUCTORS IN ARMATURE SLOTS

§ 6. **Lap Winding.**—Whichever type of winding be adopted there are usually two or more (an even number) conductors per armature slot to facilitate the end connections.

Imagine the armature to consist of a cylinder of cardboard, and to be cut along a line parallel to the axis, and folded out flat, or developed, as it is called. We then obtain a view somewhat as shown in Fig. 66, where the unshaded portions represent the slots.

Suppose now we connect the end of a conductor (a) under a north pole to the end of a conductor (b) under a south pole (*see* Fig. 67), we get an addition of E.M.F.'s.

Let these conductors be a distance apart equal to the polar pitch (distance from mid-point of one pole

to mid-point of the next measured round the armature). Let now the other end of (b) be connected to a conductor (c) next to (a)—either just before it or just

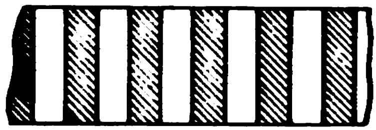

FIG. 66.—PORTION OF "DEVELOPED" ARMATURE

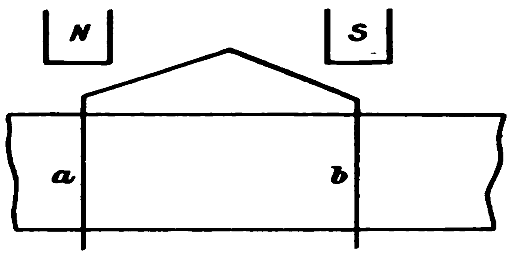

FIG. 67.—UPPER AND LOWER CONDUCTORS CONNECTED

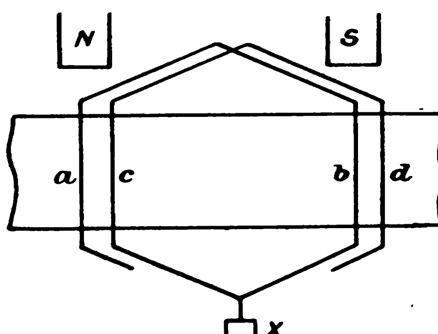

FIG. 68.—METHOD OF FORMING "LAP" WINDING

after it. Connect the other end of (c) to the conductor (d) next to (b) a pole pitch away. (*See* Fig. 68.)

Proceeding thus we obtain what is known as a lap winding.

The number of conductors is so chosen that, after proceeding all round the armature, we come back to the other end of the conductor (*a*) from which we started.

It should be noted that, if (*a*) and (*c*) are upper conductors, (*b*) and (*d*) will be lower ones, and vice versa. This is because the end connections may be

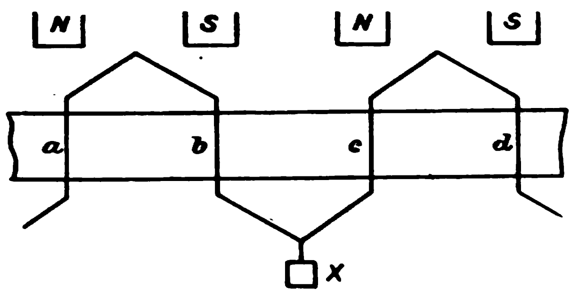

FIG. 69.—METHOD OF FORMING "WAVE" WINDING

made more easily than if they were all on the same level.

The point *X* in Fig. 68 is joined to the commutator segment corresponding to conductors (*b*) and (*c*).

§ 7. **Wave Winding.**—In this, the joint between (*a*) and (*b*) is made exactly as before, but, instead of coming back to the conductor next to (*a*), we proceed to the one under the next pole, a pole pitch away, as shown in Fig. 69.

The number of conductors is so arranged that, after proceeding once round the armature, we arrive at the conductor next to (*a*), and so on until we have used up all and arrive back at the other end of (*a*).

As before, if (*a*) and (*c*) are upper, (*b*) and (*d*) are

lower conductors, and vice versa. The point X is joined to the cummutator segment.

8. E.M.F.'s in a Lap Winding.—In all closed coil windings the algebraic sum of the induced E.M.F.'s reckoned all round the armature must vanish, as, otherwise, there would be a circulation of currents in the armature winding itself.

In Fig. 70 is represented the end view of conductors on an armature. With motion from left to right the

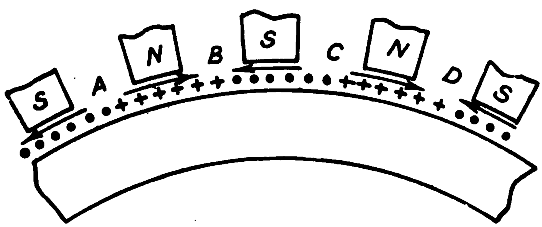

FIG. 70.—E.M.F.'s IN "LAP" WINDING

induced E.M.F.'s will be as shown. The upper conductors between A and B are in series with each other, connected through the lower conductors between B and C.

Considering only the upper conductors we see that, working from left to right, we get addition of E.M.F.'s between A and B. From B to C the addition of E.M.F.'s is in the opposite direction, and so on round the armature, with a reversal of direction between each pair of poles. This is shown diagrammatically by the arrows. Since the number of poles is even, we have an even number of changes; hence the algebraic sum vanishes.

Fig. 71 shows the case diagrammatically. We see that points A and C are of equal potential and may be connected, similarly with B and D. It must, of course, be remembered that the points $ABCD$ are fixed in

space and do not rotate with the armature. We see, then, that we may tap the conductors of a lap wound

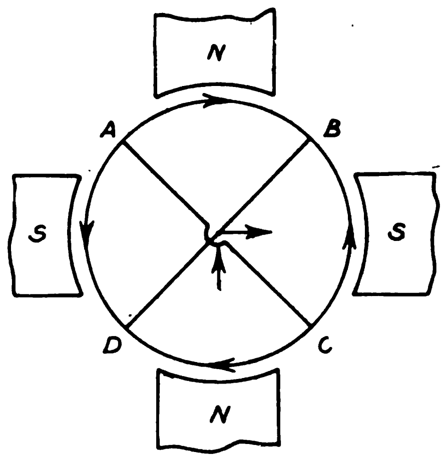

FIG. 71.—BALANCE OF E.M.F.'S IN "LAP" WINDING

armature by a number of brushes equal to the number of poles, alternate ones being connected to each other.

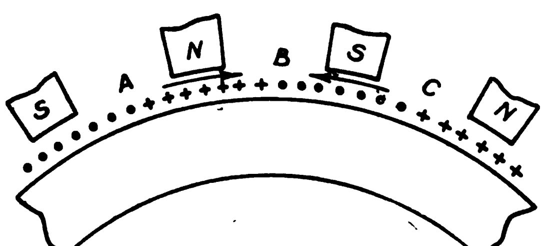

FIG. 72.—E.M.F.'S IN "WAVE" WINDING

§ 9. E.M.F.'s in a Wave Winding.—In Fig. 72 working from A to B we find that the conductors are

in series with each other, connected this time, however, by means of the lower conductors beneath all the south poles and the upper conductors beneath all the north poles. From B to C we get addition of E.M.F.'s in the reverse direction, with connections through the upper conductors beneath the south poles and the lower conductors beneath the north poles.

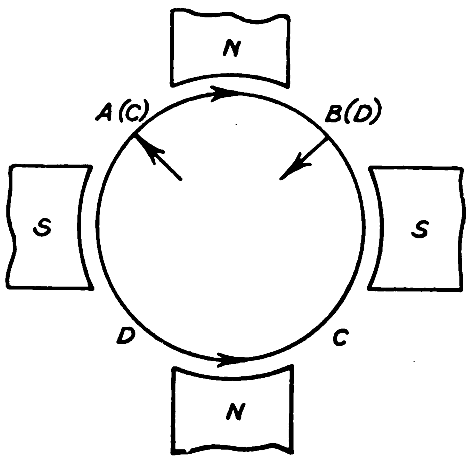

FIG 73.—BALANCE OF E.M.F.'S IN "WAVE" WINDING

All conductors have now been used up. For a wave winding, therefore, we only require to tap the conductors at two points separated from each other by a pole pitch. (*See* Fig. 73.)

Additional brushes might be placed at C and D but are unnecessary, since it is obvious that A and C are to all intents and purposes the same point separated only by one conductor, similarly with B and D.

§ **10. Winding Rules.**—From a consideration of Fig. 68, it will be obvious that any even number of

conductors may be used for a lap winding, and it will always close.

The distance between two conductors whose ends are connected together at a commutator segment (*i.e.*, (c) and (b) in Fig. 68) is known as the front pitch. The distance between two such conductors as (a) and (b) is the back pitch. These are reckoned in numbers of conductors. In a lap winding they differ by 2. (It will be remembered that there is a lower conductor between (a) and (c).)

The algebraic sum of the two pitches is known as the resultant pitch, which, in the case of a lap winding, is 2, since the pitches are in opposite directions.

In the case of a wave winding, the front and back pitches are equal if there are 2 conductors per slot.

They may differ by 2 if there are four conductors per slot, in which case it is necessary to make the resultant pitch a multiple of 4.

Now, after going once round the armature, the winding must arrive at one of the top conductors next to the one from which it started. Let $x =$ front pitch $=$ back pitch. If they are unequal, let $x =$ average pitch.

We then have

$$x \times \text{number of poles}$$
$$= \text{number of conductors} \pm 2$$

or, if there are $2p$ poles,

$$x \times 2p = N \pm 2$$

or, $$x = \frac{N \pm 2}{2p}$$

Since x must be a whole number, we see that the number of conductors $\pm$ 2 must be divisible by the number of poles.

§ **11. Total E.M.F.'s.**—Fig. 74 represents the portion of armature below two adjacent poles of a lap wound dynamo. Consider a wire at an angle θ from the line Ox. The E.M.F. induced in it will be $Blv \cos \theta$, since its velocity normal to the flux is $v \cos \theta$.

Let there be σ *upper* wires per cm. of periphery.

In an arc $d\theta$ there will be $\sigma r d\theta$ wires in series with each other connected through a similar number of

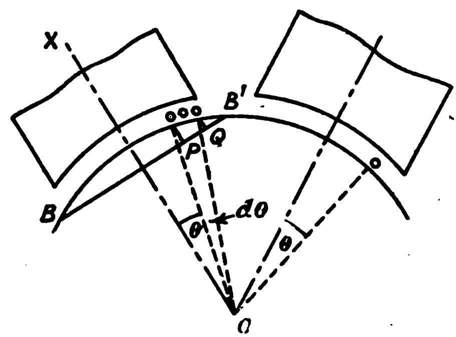

FIG. 74.—CALCULATION OF DYNAMO E.M.F.

lower wires underneath the adjacent pole. Hence, the E.M.F. for these wires will be

$$2Blv \cos \theta \, \sigma r d\theta$$

Hence the total E.M.F. from B to B' (the brushes) will be

$$\int_{B}^{B'} 2Blv \cos \theta \, \sigma r d\theta = 2v\sigma \int_{B}^{B'} Blr \cos \theta \, d\theta$$

Now we see that $rd\theta \times \cos \theta = PQ$

Hence, assuming flux to be, at all points between B and B' parallel to XO

$\therefore$ $Blr \cos \theta \, d\theta =$ amount of flux cut by the portion of armature in the arc $d\theta$.

$$\therefore \quad \int_B^{B'} Blr \cos\theta \, d\theta = \text{polar flux } \Phi$$

$$\therefore \text{ E.M.F.} = E = 2\Phi v\sigma$$

$$\text{If} \quad N = \text{total wires}$$

$$\frac{N}{2} = \text{total } \textit{upper} \text{ wires}$$

$$= 2\pi r\sigma \quad \therefore \quad \sigma = \frac{N}{4\pi r}$$

$$\text{Also} \quad v = 2\pi rn$$

$$\therefore \quad E = 2\sigma \times 2\pi rn \times \frac{N}{4\pi r}$$

$$= \Phi Nn \times 10^{-8} \text{ volts.}$$

If the armature be wave wound there will be $2p\sigma r\, d\theta$ upper and lower wires in series in the arc $d\theta$, since there are $\sigma rd\theta$ under each pole.

Hence, the total E.M.F. in the arc will be

$$2pBlv \cos\theta \, \sigma rd\theta$$

Integrating between B and B' we have used up all the wires, and, proceeding exactly as before, we see that the E.M.F. E now comes to be

$$p\Phi Nn \times 10^{-8} \text{ volts.}$$

We may combine these two formulae in one. Let y be the number of parallel paths for armature current. Then we may say

$$E = \frac{2p\Phi Nn \times 10^{-8}}{y} \text{ volts.}$$

Since, for lap wound armatures, $y = 2p$, and for wave wound, $y = 2$, this formula gives us those which we have just proved.

§ 12. Necessity for Good Insulation.—It will be obvious from Figures 68 and 69 that the conductor in the same slot with (a) will be connected with the one

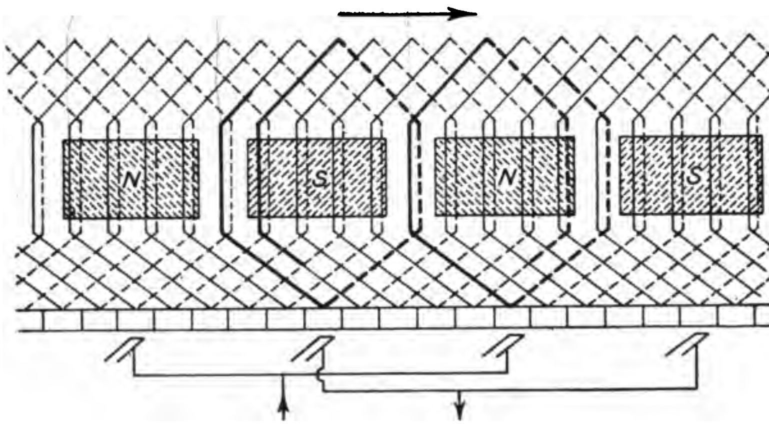

FIG. 75.—" DEVELOPED " 4-POLE, 40 WIRE, " LAP " WOUND ARMATURE

Armature motion in direction of arrow
Poles assumed below surface of paper

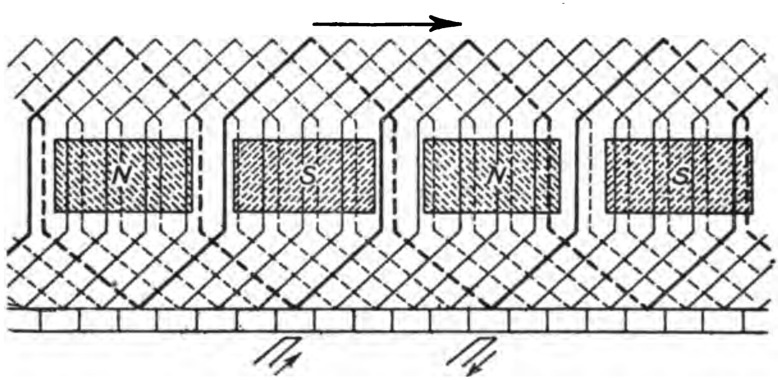

FIG. 76.—" DEVELOPED " 4-POLE, 38 WIRE, " WAVE " WOUND ARMATURE

Armature motion in direction of arrow
Poles assumed below surface of paper

a pole pitch away in the reverse direction from the one to which (*a*) is connected, and the corresponding commutator segment will be a pole pitch from X; hence, the maximum P.D. must occur periodically between each pair of conductors in any one slot.

It is, therefore, highly necessary that there should be very good insulation between conductors.

With a ring wound armature it is obvious, of course, from a consideration of Fig. 64 that there is not the same necessity, since the P.D. between any two neighbouring conductors is only equal to the E.M.F. generated in one conductor. Fig. 75 shows a 4 pole, 40 wire, lap wound, and Fig. 76 a 4 pole, 38 wire, wave wound armature.

Examples

(1) A 4 pole generator having 500 peripheral conductors runs at 300 revs. per min., and develops an E.M.F. of 440 volts. Is it a lap or a wave wound machine? Find the polar flux.

(2) The pole cores of a 6 pole, lap wound, machine have a cross sectional area of 152 sq. cms. The armature carries 360 conductors and runs at 800 revs. per min. If the air gap flux per pole is $1{\cdot}84 \times 10^6$ C.G.S. lines, find the E.M.F. produced.

What is the value of B in the pole cores if the leakage coefficient is 1·34?

(3) If the machine in Question (2) had 362 conductors, and were wave wound, what E.M.F. would be produced?

CHAPTER VII

EXCITATION AND THE MAGNETIC CIRCUIT

§ 1. **Methods of Excitation.**—We have seen that, at first, permanent magnets were used to provide the field required for producing E.M.F. This method is now, however, no longer used, except in very small machines, such as petrol engine ignition magnetos, as a permanent magnet gradually loses its strength and, in any case, is never so strong as an electrically excited one. There are four methods of excitation in use, known as—

(*a*) Separate.
(*b*) Series.
(*c*) Shunt.
(*d*) Compound.

Whichever method is adopted, coils of wire are wound round each of the field magnets and a current of electricity is passed through these coils, thus providing the necessary magnetic field.

(*a*) The coils surrounding field magnets are connected to a separate source of supply, such as a storage battery or another dynamo. This is shown diagrammatically in Fig. 77.

(*b*) The whole current supplied by the machine passes through the field coils, which consist of a few turns in series with the armature brushes.

(*c*) A small portion of the current supplied by the armature flows through the field coils, which consist of a large number of turns "shunted" across the armature brushes.

(*d*) A combination of methods (*b*) and (*c*). There are two separate field windings, one in series with,

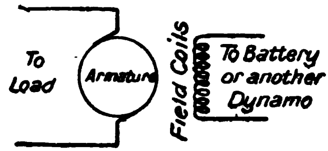

FIG. 77.—SEPARATELY EXCITED FIELD

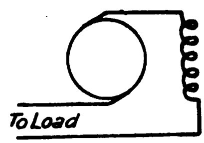

FIG. 78.—SERIES WOUND FIELD

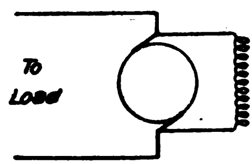

FIG. 79.—SHUNT WOUND FIELD

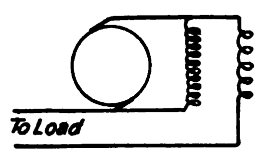

FIG. 80 (*a*).—" SHORT SHUNT " COMPOUND FIELD

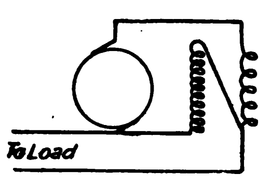

FIG. 80 (*b*).—" LONG SHUNT " COMPOUND FIELD

and one shunted across, the armature. There are two methods of doing this, the shunt coils being either connected directly across the brushes, as in Fig. 80(*a*), which is known as a short shunt, or else connected across the armature and series coils combined, as in Fig. 80(*b*). This is known as a long shunt.

Except with method (*a*) it might be argued that, since there is no current at starting, there will be no field, and hence no E.M.F. will be produced.

This is incorrect, however, because iron always retains a small amount of magnetization which is just sufficient to produce a small E.M.F. in the armature, which sends a small current through the field coils. This small current strengthens the field and produces a greater E.M.F., and so on until the machine is generating the E.M.F. for which it is designed. Further, when the machine is supplying its maximum current, the induction in the iron is just below saturation.

The behaviour of the different types and their uses will be reserved until we consider characteristics in Chapter IX.

§ 2. **Parts of a Dynamo.**—A direct current dynamo has for its main parts—

(*a*) The armature winding.

(*b*) The armature core on which the winding is placed.

(*c*) The field magnets or poles.

(*d*) The field coils for exciting the poles.

(*e*) The iron yoke, from which the poles project radially.

Fig. 81 shows the various parts diagrammatically.

In order to reduce the reluctance of the magnetic path as much as possible, the space between the armature and the poles, which is known as the air gap, should be as small as is permissible from a manufacturing

point of view, and it is chiefly for this reason that the windings of the armature are embedded in slots formed in the core. Another reason is that a positive drive is given to the conductors by the slot walls.

The armature core is made of iron of good permeability so as to offer, as before, small reluctance to the

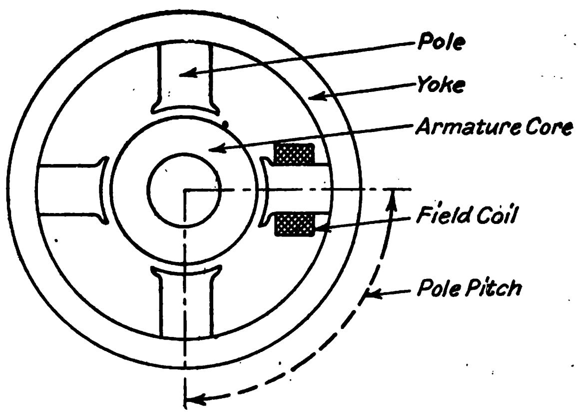

FIG. 81.—MAIN PARTS OF A DYNAMO

magnetic path. It is constructed of thin flat circular stampings or laminations insulated from each other by thin sheets of paper, mica, or insulating varnish. The laminations are usually of the order of 1 mm. thick. The reason for this is, that, as the armature rotates, the iron core itself cuts lines of magnetic flux and thus has E.M.F.'s generated in it which cause local currents, called eddy currents, to circulate.

These currents, of course, form a source of loss, producing heat in the armature, and it is with the object of reducing them to a minimum that the core is laminated.

§ 3. Variation of Eddy Currents with Number of Laminations.—Consider a plate whose length is great

compared with its thickness x, and rotating so that its dimension x is normal to the flux. The induced E.M.F. e will be proportional to x, and the resistance to eddy currents will be inversely proportional to x. In Fig. 82a the E.M.F. will be e, and the current will be ex, since the resistance is $\frac{1}{x}$. Hence, the loss is e^2x.

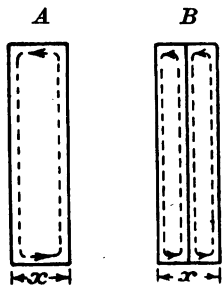

FIG. 82.—EDDY CURRENTS IN LAMINATIONS

In Fig. 82b the E.M.F. per plate is $\frac{e}{2}$. The resistance per plate is $\frac{2}{x}$. Hence, the current is $\frac{e^2x}{4}$ and the loss is $\frac{e^2x}{8}$. But there are two plates, so the total loss is $\frac{e^2x}{4}$, or, in general, if there are n plates, the total loss will be $\frac{e^2x}{n^2}$.

In other words, the eddy current loss varies inversely as the square of the number of plates.

§ **3a. Magnetic Path.**—The dotted lines in Fig. 83 show the mean paths of magnetic flux in a machine. The extensions at the bottom of the poles, called shoes or polar horns, are in order to provide for what is known as "fringing." The flux does not leave the pole perfectly normal to its face, but somewhat as shown in Fig. 84.

It becomes necessary to know how to calculate the ampere turns per pole required to produce the necessary flux. The ampere turns on pole N have to drive half the polar flux round the path CBA and half round $C'B'A'$.

The ampere turns on S and S' supply the magneto

motive force for the remaining halves of the paths. Hence, we may say that the total magneto-motive-force per field coil is equal to the product of the total polar flux into the reluctance of the half path *CBA*.

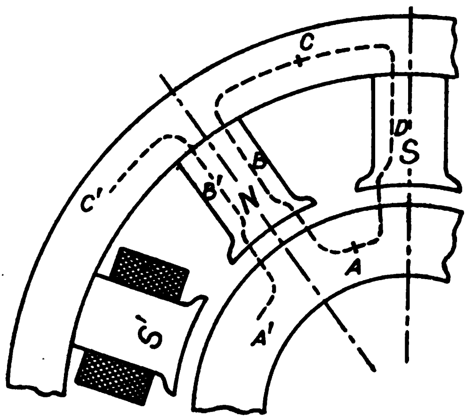

FIG. 83.—PATHS OF FLUX IN A MACHINE

The magnetic circuit may be considered to be made up of the following components—

(*a*) The yoke.
(*b*) The pole core.
(*c*) The air gap.
(*d*) The armature teeth.
(*e*) The iron part of the armature.

The requisite magneto motive force or ampere turns must be calculated separately for each of those portions.

Machines are designed so that the various inductions are below the saturation point, so that the loss due to leakage will not be great.

Working values of *B* are usually those just below

the knee of the *BH* curve, and the following table gives, roughly, the intensities used in practice—

		Saturation.	Working Induction.
Magnets	Wrought iron . .	21,000	16,500
	Cast steel . . .	20,000	15,500
	Cast iron . . .	14,000 12,000	8,500 7,500
Armature	Sheet iron and steel	21,000	18,000 teeth 12,500 core

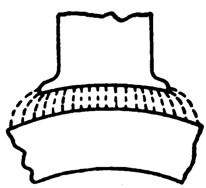

FIG. 84.—" FRINGING " AT POLAR HORNS

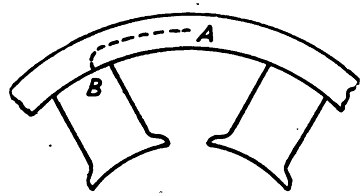

FIG. 85.—FLUX IN YOKE

Air gap intensities vary from 4,000 to 7,000.

These figures are only rough, and are often exceeded. We see, then, that once we have fixed the dimensions of the various components we can, from a knowledge of the flux required, calculate the necessary ampere turns per pole.

Ampere turns required for the various parts of an actual machine taken as an example, are—

Yoke	480	5·5%
Pole core	780	8·8%
Air gap	7,200	82·0%
Teeth	40	·5%
Armature core. . .	290	3·2%
	8,790 Total	

§ 4. Ampere Turns for Yoke and Pole Core.—If Φ is the total useful polar flux penetrating the armature, the flux through the yoke and pole is $\gamma\Phi$ where γ is the leakage factor. (*See* Chap. III.)

The flux through portion AB of the yoke (Fig. 85) is half the total polar flux $= \frac{1}{2}\gamma\Phi$. Dividing this by the cross section, we can obtain B and thence μ from the curves for the iron.

The magneto motive force required may now be calculated from the formula—

$$\text{M.M.F.} = \frac{\frac{1}{2}\gamma\Phi \times \text{length of mean path } AB}{\mu \times \text{cross section of yoke}}$$

Now, $\dfrac{\frac{1}{2}\gamma\Phi}{\text{Cross section}}$ is the induction B_y in the yoke,

and $\dfrac{B_y}{\mu}$ is the corresponding H_y found from the curve.

$$\text{Hence, amp. turns} = \frac{10}{4\pi} \times \text{M.M.F.}$$

$$= \frac{10}{4\pi} \times H_y \times AB.$$

To calculate the M.M.F. required for the pole core, we may neglect the effect of the polar horns, since the increase in length of path of flux at C (Fig. 86) is compensated for by the increase in cross sectional area. As before we can calculate B_p by dividing Φ by the cross sectional area XX', whence we obtain H_p. Hence,

$$\text{Amp. turns} = \frac{10}{4\pi} \times \text{M.M.F.}$$

$$= \frac{10}{4\pi} \times \frac{\gamma\Phi \times AB}{\mu \times \text{cross section } XX'}$$

which, as before,

$$= \frac{10}{4\pi} \times H_p \times AB.$$

§ 5. **Ampere Turns for Air Gap.**—This problem is not so straightforward as the two preceding ones, as three factors have to be considered. These are—

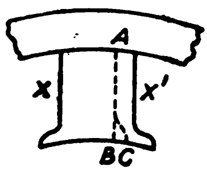

Fig. 86.—Effect of Polar Horn on Flux

(*a*) The fringing effect already described is equivalent to widening the air gap included under the pole. The amount depends upon the size of interpolar arc and the length of the gap. The subject has been investigated by F. W. Carter, and the following is an abstract from tables which have been compiled showing the fraction of gap length which must be added to the gap breadth for various ratios of

$$\frac{\text{Interpolar arc}}{\text{gap length}}$$

$r = \dfrac{\text{Interpolar arc}}{\text{gap length}}$	10	20	30	40	50	60
c = Fraction of gap length to be added to gap breadth .	2·43	3·28	3·78	4·14	4·40	4·66

Thus, if the gap breadth is 50 mms., the interpolar arc 20 cms., and the gap length 1 cm.,

$$r = \frac{20}{1} = 20 \qquad \therefore c = 3{\cdot}28$$

∴ Total equivalent breadth

$$= 50 + 3{\cdot}28 \times 1 = 53{\cdot}28 \text{ cms.}$$

In addition to this, there will be fringing at the ends of the poles where the armature protrudes beyond them. The effect of this will be to increase the length of armature under the poles.

The amount will depend upon l' (Fig. 87), the extent to which it protrudes beyond the poles.

As before, the amount to be added is the air gap g multiplied by a definite factor d.

The following table gives values of d for various values of $\frac{l'}{g}$

$\frac{l'}{g}$	1	2	3	4	5	6	7	8
d	1·2	1·9	2·4	2·75	3·1	3·25	3·5	3·65

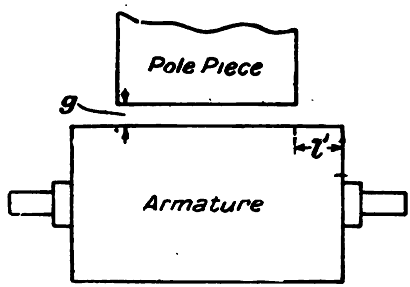

FIG. 87.—ARMATURE EXTENDING BEYOND POLE

If, therefore, the length of armature under the pole is l, the *total* equivalent length is $l + dg$.

(*b*) The mean length of the actual path in the air gap is difficult to decide, owing to the fact that lines of flux do not all enter the tooth normally, but some penetrate some distance into the slots and then enter the walls of the teeth. This is equivalent to a lengthening of the air gap.

Assuming that the lines between the teeth describe quadrants (see Fig. 88), calculations have been made and tables compiled giving the correction coefficient k_1 by which the air gap length must be multiplied for various relative values of gap length, tooth width,

and slot width. Hence, if the gap length is g, the equivalent gap length is $k_1 g$; k_1 varies between about 1·1 and about 1·2.

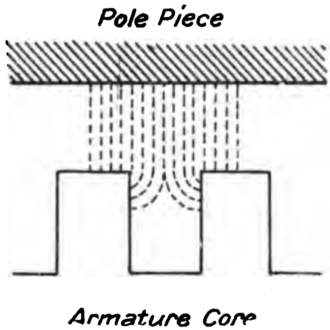

FIG. 88.—FLUX IN AIR GAP

A simple and not very inaccurate formula is given by A. Müller,

$$k_1 = \frac{1 + 4\frac{g}{s}}{\frac{t}{t+s} + 4\frac{g}{s}}$$

where t = width of tooth
s = width of slot
g = gap length

(c) In most armatures spaces are left at intervals along the axial length with no armature plates. The purpose of this is ventilation. The flux passes into the sides of the plates in the intervals just as it does between teeth, and, therefore, a further correction k_2 is required for the gap length. Müller's formula above may be used if, for s, we substitute the width y of the duct, and, for t, we substitute x, the distance between

ducts. The values of k_2 are usually more equal to unity than k_1.

The corrected gap length now becomes $k_1 k_2 g$.

Taking into account the above three corrections, we now find that the equivalent air gap reluctance is

$$\frac{k_1 k_2 g}{(b + cg)(l + dg)}.$$

where b = polar arc
l = length of armature under poles.

Knowing Φ we can, as before, calculate the ampere turns for the air gap.

$$\begin{aligned} \text{Amp. turns} &= \frac{10}{4\pi} \times \text{M.M.F.} \\ &= \frac{10}{4\pi} \times \Phi \times \frac{k_1 k_2 g}{(b + cg)\ (l + dg)} \\ &= \frac{10}{4\pi} \times H_g \times k_1 k_2 g, \text{ since } H = B \text{ in air.} \end{aligned}$$

§ 6. Ampere Turns for Teeth.—In most armatures the walls of the slots are parallel and, therefore, the teeth taper inwards as shown in Fig. 89. We assume that all the flux enters the top of the tooth and none in the walls. The induction intensity will therefore increase as we pass inwards. We obtain the flux per tooth by dividing the total polar flux Φ by the number of teeth which receive the flux. This number is about 10 per cent. more than the actual number of teeth covered by the pole on account of fringing.

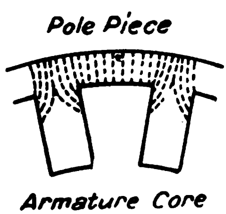

FIG. 89.—SHOWING SHAPE OF TEETH

To obtain the induction at the top of the tooth it is necessary to divide the flux per tooth by the area of iron at the top of the tooth. This is less than the

total area of the top of the tooth, on account of the insulation between laminations, which usually amounts to about 13 per cent. of the total. Hence, if l is the total length of laminations, excluding ventilating ducts, the length of iron will be ·87 l and if t_1 is the thickness of the top of the tooth, ·87 lt_1 will be the area of iron receiving flux.

Hence, induction at top of tooth is given by

$$B_{t1} = \frac{\text{Flux per tooth}}{\cdot 87 lt_1}$$

From our tables we can find H_{t1} corresponding to this.

Similarly, we can find B_{t2}, the induction at the bottom of the tooth from the equation.

$$B_{t2} = \frac{\text{Flux per tooth}}{\cdot 87 lt_2}$$

Where t_2 is the thickness of the bottom of the tooth.

We again obtain the corresponding H_{t2}.

We may assume H to vary uniformly down the tooth and take $\frac{H_{t1} + H_{t2}}{2}$ as the mean H for the tooth. This is not strictly true on account of the slight curvature of the BH curve for the iron, but the error is not great for practical purposes.

The ampere turns will now be given by

$$\text{Amp. turns} = \frac{10}{4\pi} \times \text{M.M.F.}$$

$$= \frac{10}{4\pi} \times \frac{\text{Flux per tooth} \times \text{height of tooth}}{\text{Mean permeability} \times \text{mean area}}$$

$$= \frac{10}{4\pi} \times \frac{\text{Mean } B \times \text{height of tooth}}{\text{Mean permeability}}$$

$$= \frac{10}{4\pi} \times \text{mean } H \times \text{height of tooth.}$$

§ 7. **Ampere Turns for Armature Core.**—The flux along the path AB (Fig. 90) through the armature core is equal to half the polar flux. To obtain the induction B_c we must divide this figure by the cross sectional area of the iron at XX'. As before, allowance must be made for the ventilating ducts and for the insulation between plates.

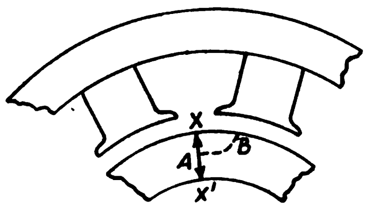

FIG. 90.—FLUX IN ARMATURE CORE

Having obtained B_c, H_c is found from the curves for the iron, and the equation for ampere turns becomes

$$\text{Amp turns} = \frac{10}{4\pi} \times \text{M.M.F.}$$

$$= \frac{10}{4\pi} \times H_c \times \text{mean length of path } AB.$$

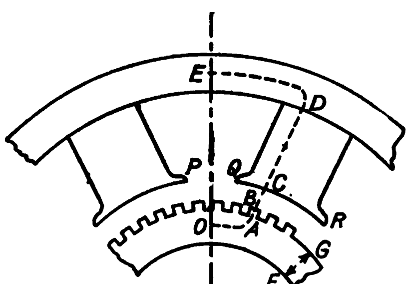

FIG. 91.—CALCULATION OF FIELD AMPERE TURNS

Numerical Example

In order to illustrate the matter dealt with in the preceding paragraphs, we shall work out a fairly complicated example taken from an actual machine.

A 6 pole lap wound machine with 360 conductors is designed to run at 800 revs. per min. and produces 88·4 volts E.M.F. Referring to Fig. 91, the various dimensions are as follows—

$$\begin{aligned} OA &= 8 \text{ cms.} \\ AB &= 1{\cdot}7 \text{ cms.} \\ BC &= {\cdot}35 \text{ cm.} \\ CD &= 16 \text{ cms.} \\ DE &= 18 \text{ cms.} \\ PQ &= 7{\cdot}9 \text{ cms.} \\ QR &= 14{\cdot}6 \text{ cms.} \end{aligned}$$

The cross section of the yoke is 97·6 cms., and that of the poles is 152 sq. cms. The poles are 15·9 cms. long in the direction of the axis of the armature.

The armature core is 17·9 cms. long in an axial direction and has two ventilating ducts 5·3 cms. apart, each 1 cm. wide.

The slots have parallel walls and the teeth are ·77 cm. wide at the top and ·66 cm. wide at the bottom. The armature stampings were made of Lohlys steel. The leakage factor of the poles may be taken as 1·34.

The flux per pole entering the armature is given by

$$E = \Phi N n \times 10^{-8}$$

$$\text{or,} \quad \Phi = \frac{88{\cdot}4 \times 10^{8} \times 60}{360 \times 800}$$

$$= 1{\cdot}84 \times 10^{6} \text{ C.G.S. lines.}$$

(1) Yoke. The flux in the yoke is half the polar flux, which is equal to $1{\cdot}34 \times 1{\cdot}84 \times 10^{6}$

$\therefore$ Flux in yoke

$$= 1{\cdot}26 \times 10^6 \text{ lines}$$

The cross section

$$= 97{\cdot}6 \text{ cms.}^2$$

$\therefore$ Induction

$$B_y = 11{,}900$$

From the iron curve we find

$$H_y = 8{\cdot}8$$

The length of the path ED is 18 cms.

$\therefore$ Amp. turns required

$$= \frac{10}{4\pi} \times 8{\cdot}8 \times 18$$

$$= 116.$$

(2) Pole Core. The induction

$$B_p = \frac{1{\cdot}34 \times 1{\cdot}84 \times 10^6}{152}$$

$$= 16{,}200$$

We find $H_p = 25$

The length of path

$$CD = 16 \text{ cms.}$$

$\therefore$ Amp. turns

$$= \frac{10}{4\pi} \times 25 \times 16$$

$$= 320.$$

(3) Air Gap. The breadth of the air gap is 14·6 cms. The inter polar arc is 7·9 cms., and the air gap height is ·35 cm. Hence, ratio

$$\frac{\text{inter polar arc}}{\text{air gap}} = \frac{7{\cdot}9}{{\cdot}35} = 22{\cdot}5$$

The value of C for this is, from the first table in § 5, 3·39.

∴ The equivalent air gap breadth

$$= 14 \cdot 6 + 3 \cdot 9 \times \cdot 35$$
$$= 14 \cdot 6 + 1 \cdot 135$$
$$= 15 \cdot 79 \text{ cms.}$$

The length in an axial direction is 15·9 cms. The armature protrudes 1 cm. beyond the pole, hence the ratio

$$\frac{l'}{g} = \frac{1}{\cdot 35} = 3$$

∴ From the second table in § 5,

$$d = 2 \cdot 4$$

Hence, the equivalent length in an axial direction

$$= 15 \cdot 9 + 2 \cdot 4 \times \cdot 35$$
$$= 15 \cdot 9 + \cdot 84$$
$$= 16 \cdot 74 \text{ cms.}$$

Thus, the area

$$= 15 \cdot 78 \times 16 \cdot 74 = 264 \text{ sq. cms.}$$

The length of gap along the path of flux = ·35. Using Müller's formula, we find

$$k_1 = 1 \cdot 185$$

For the ventilating ducts, again using Müller's formula, we have

$$k_2 = 1 \cdot 07$$

∴ Equivalent length of path

$$= 1 \cdot 07 \times 1 \cdot 135 \times \cdot 35$$
$$= \cdot 44$$

Now flux per pole through air gap

$$= 1 \cdot 84 \times 10^6$$

$$\therefore \quad B_g = H_g = \frac{1 \cdot 84 \times 10^6}{264}$$
$$= 6{,}960$$

$$\therefore \text{ Amp. turns } \frac{10}{4\pi} \times 6{,}960 \times \cdot 44$$
$$= 2{,}440.$$

(4) TEETH. There are $\frac{14 \cdot 6}{\cdot 77 \times \cdot 7} = 10$ teeth under cover of the pole, therefore, increasing this number by 10 per cent. we say that 11 teeth receive the flux $1 \cdot 84 \times 10^6$.

Now the gross length of armature laminations is 15·9 cms. Allowing for insulation, we have

Length of iron
$$= \cdot 87 \times 15 \cdot 9$$
$$= 13 \cdot 8 \text{ cms.}$$

$\therefore$ Area of top of tooth
$$= \cdot 77 \times 13 \cdot 8 \text{ cms.}^2$$

Induction at top of tooth
$$B_{t1} = \frac{1 \cdot 84 \times 10^6}{11 \times \cdot 77 \times 13 \cdot 8}$$
$$= 15{,}750$$

whence $H_{t1} = 18$

Area at bottom of tooth
$$= \cdot 66 \times 13 \cdot 8 \text{ cms.}^2$$
$$\therefore \quad B_{t2} = \frac{1 \cdot 84 \times 10^6}{11 \times \cdot 66 \times 13 \cdot 8}$$
$$= 18{,}400$$

It will be noticed that this is rather on the high side. We obtain

$$H_{t2} = 110$$
$$\therefore \quad \frac{H_{t1} + H_{t2}}{2} = \frac{18 + 110}{2} = 99$$
$$\text{Amp. turns} = \frac{10}{4\pi} \times 99 \times 1 \cdot 7$$
$$= 134.$$

(This is very much higher than would be expected, and is due to the fact that the induction is high and on the horizontal portion of the BH curve.)

(5) ARMATURE CORE. The length of iron in the armature core is the same as the teeth (*i.e.*, 13·8 cms.). The radial depth

$$= 6{\cdot}3 \text{ cms.}$$

Hence, the cross section

$$= 6{\cdot}3 \times 13{\cdot}8 = 87 \text{ cms.}^2$$

$$\text{The flux} = {\cdot}92 \times 10^6$$

$$\text{Induction } B_c = \frac{{\cdot}92 \times 10^6}{87}$$

$$= 10{,}600$$

$$\text{Whence } H_c = 5$$

$$\text{The path } AO = 8 \text{ cms.}$$

$$\therefore \text{ Amp. turns} = \frac{10}{4\pi} \times 5 \times 8$$

$$= 32.$$

Thus, the total ampere turns required per pole

$$= 116 + 320 + 2{,}440 + 134 + 32$$

$$= 3042.$$

EXAMPLES

(1) A compound "long shunt" machine is delivering 200 amps. at 100 volts terminal P.D. The armature resistance is ·015 ohm, the series field coil is ·015 ohm, and the shunt field coil 30 ohms. Find the E.M.F. which the machine is producing, and the shunt current.

Other things being equal, what would be the value of the shunt current if it were a short shunt machine?

(2) In a certain machine for which Fig. 91 represents the magnetic circuit, the dimensions are as follows: OA = 7·5 cms., cross section = 373 cms.^{2}, BC = ·6 cms.,

air gap circumferential breadth under polar arc=25 cms., CD = 20 cms., cross section = 900 cm.^{2}, DE= 22·5 cms., cross section = 450 cms.^{2} The length of poles in a direction parallel to the armature axis is 40 cms. The axial length of armature is 46 cms., and there are four ventilating duct, each 1·5 cms. wide, equally spaced along it.

The teeth are 1·1 cm. wide at the top, and the slots are 1 cm. wide. The ratio of inter polar arc to gap length is 10. Find the equivalent length and cross section of the air gap.

(3) In Question (2), neglecting the teeth and taking the cross sections given above as those of the iron, exclusive of insulation between laminations, find the total air gap flux per pole if the pole core induction is B = 13,000, the leakage coefficient being 1·3, and also the ampere turns per pole if the corresponding H in the pole cores and yoke is 18. The permeability of the armature core may be taken as equal to that of the pole cores.

CHAPTER VIII

COMMUTATION AND ARMATURE REACTION

§ 1. Change of Direction of Current in Windings.— It has been pointed out in Chapter VI that the currents induced in the conductors of an armature change in direction as they pass from the influence of one pole to the next. The commutator forms the means whereby these currents are collected and given to the outer circuit in a constant direction. The change in direction of the current as the conductor passes from one pole to another, or, what is the same thing, as the corresponding commutator segment passes across the brush, is called commutation.

In Fig. 92, *A* and *B* represent, diagrammatically, loops formed by armature conductors. They are, respectively, the loops consisting of the broken and unbroken lines in Fig. 93, but have been separated apart for the sake of clearness.

Fig. 93 shows a lap winding ; but exactly the same holds for a wave winding, the loops in that case extending right round the armature. In the case of a "coil" winding they will each, of course, consist of several turns. *C* and *D* are commutator segments, and the rotation of the armature and commutator is assumed to be from right to left. The brush will always be wide enough to make contact with at least two, and sometimes more, commutator segments.

The arrows show the direction of flow of current, and it is seen that, in the coil *B* before segment *C* is in contact with the brush, the current is from left to right.

In Fig. 94, commutator segment *C* is commencing

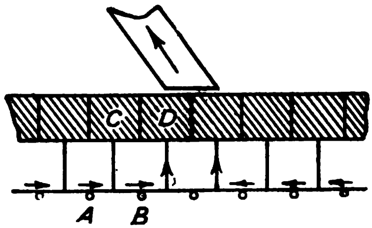

FIG. 92.—DIRECTION OF CURRENTS AT BRUSH CONTACT

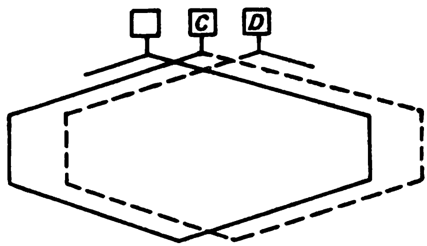

FIG. 93.—LOOPS FORMED BY CONDUCTORS

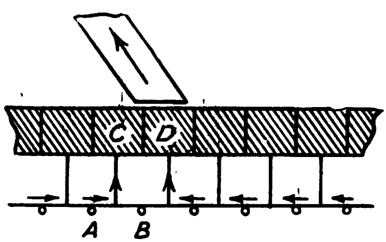

FIG. 94.—COMMENCEMENT OF COMMUTATION IN LOOP B

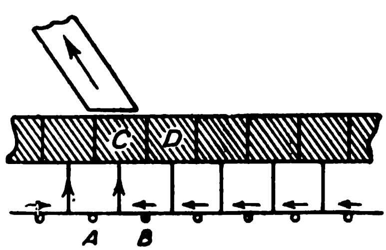

FIG. 95.—COMPLETION OF COMMUTATION IN LOOP B

contact with the brush, and some of the current can now pass through C. The current through B is, accordingly, decreasing.

When the brush covers equal amounts of C and D, equal currents will flow to the brush through C and D with ideal commutation (*i.e.*, these from the left and from right respectively). The current through B will now be zero.

As the toe of the brush further recedes from D, less current from the right can now pass through it, and some will pass through B, from right to left, and will find its way to the brush through C.

This goes on until the toe of the brush has left segment D (Fig. 95). The current in B must now be the full current from right to left.

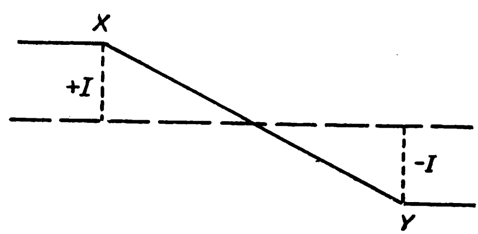

FIG. 96.—IDEAL CURRENT VARIATION DURING COMMUTATION

§ 2. **Ideal Commutation.**—Assume that the current changes its value at a constant rate. If a curve be plotted showing current and time, it will be in the form shown in Fig. 96, where horizontal distance represents time and vertical distance represents current. It will be seen that the change from $+I$ to $-I$ is a straight line.

The point X is the instant at which the heel of the brush commences contact with segment C, and Y is where the toe of the brush leaves D. The amount of current which passes to the brush through C is equal

to the drop in current passing through B. Now, since the current varies directly as the time and the rate at which the brush increases its contact with C and decreases its contact with D is also proportional to time, it becomes clear that the current distribution over the brush surface remains uniform. It may be shown mathematically that this even distribution of current entails the least possible resistance loss and, therefore, least possible heating at the contact. If the current density were not uniform, those portions where the current density were greater would become more heated, and in practice that is what happens. It becomes necessary, therefore, to consider how commutation departs from the ideal and what means there are of preventing it from so doing.

§ 3. **Reactance E.M.F.**—The current flowing in the coil or coils undergoing commutation will itself give rise to lines of magnetic flux. As the current changes, an E.M.F. will be induced in these coils, owing to the change of self-produced flux which is linked with them.

Now, the direction of this E.M.F. will be such as to oppose the change of current, and hence the current will be prevented from changing so rapidly as it should do.

Fig. 97 shows this state of affairs. The full line represents the ideal commutation discussed in § 2, and the dotted line shows the effect of the back E.M.F., or reactance E.M.F., as it is called, in preventing the current from changing sufficiently rapidly.

It will be seen that the change is gradual at first, but then becomes more rapid as the area of the receding commutator segment in contact with the brush becomes less and less, At the end of the period of short circuit the current will not be at Y but at some such point as Y'.

The effect of this is twofold. First, the current

distribution is no longer uniform, and there is a greater density at the toe of the brush. This gives rise to local heating at the brush contact.

Second, the current has to change more rapidly as the commutator segment leaves the brush, and this occasions a high induced E.M.F. in the coil which will cause some sparking when the contact is broken at Y', and thus still further heat the already unduly heated surface.

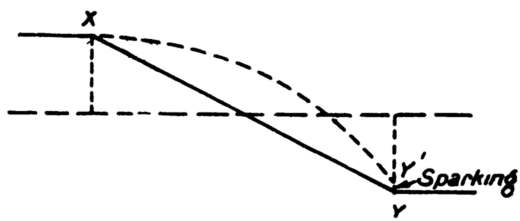

FIG. 97.—EFFECT OF REACTANCE VOLTAGE ON COMMUTATION

It will be readily seen that, in course of time, the commutator surface will wear rapidly and still further enhance the trouble.

§ 4. Methods of Overcoming Bad Commutation.—There are three methods of mitigating against the evils described above. These are—

(*a*) Giving brushes a lead.
(*b*) Use brushes with large contact resistance.
(*c*) Inter-poles.

(*a*) This is now more frequently used in small than in large machines, but 20 years ago it was the only method of practical importance in use. It has been stated that the trouble is due to the reactance voltage preventing the current from changing properly.

If, then, it is possible to impress upon the loops undergoing short circuit an E.M.F. in the opposite

direction and equal to the reactance E.M.F., the current will be free to change in accordance with the straight line in Fig. 96. The effect of giving the brushes a lead is to delay commutation until the conductors of the loops in question are partially under the influence of the next pole, and are, therefore, cutting flux which tends to produce current in the opposite direction (*i.e.*, to help the current to change).

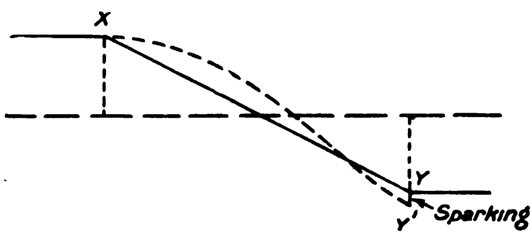

FIG. 98.—EFFECT OF TOO MUCH BRUSH "LEAD"

It will be readily seen that too much lead will cause the current to change too rapidly and will produce the result shown in Fig. 98.

Y′ Y now represents a condition of sparking in the reverse direction from that shown in Fig. 97.

The amount of lead required will clearly depend upon the value of the reactance voltage which, in turn, depends upon the armature current. Lead must accordingly be varied with the load which naturally is, on big machines, a serious inconvenience.

(*b*) If the resistance at the brush contacts be high in comparison to the reactance E.M.F., the latter will have little effect on the current collected by the brush, which will, therefore, be nearly proportional to the area of commutator segment with which contact is being made. Hence, as explained in § 2, the current variation will more nearly approach the ideal straight

line. Carbon brushes are now almost invariably used on big machines, since they fulfil these requirements.

The old copper gauze brushes used to give about ·035 volt drop at each brush. Carbon brushes give between 1 and 1½ volts drop per brush. The permissible current density with copper is about 150 amps. per square inch of contact, whereas, with carbon it is in the neighbourhood of 45 or 50, depending upon the quality of the carbon.

It is evident, then, that the brushes, brush gear, and commutator must be larger with carbon than with copper brushes, but the additional expense of this, together with the higher loss at brush contacts, is more than compensated by the fact that the lead may be fixed and will give sparkless running over a wide range of loads. In small machines, where the voltage drop with carbon brushes would be appreciable compared with the E.M.F. generated, copper brushes must be used.

(*c*) This method, although known for a much longer time, has only been adopted generally during the last 15 years. The idea is the same as (*a*), and consists in impressing a reversing E.M.F. upon the loops undergoing commutation. Instead, however, of giving the brushes a lead, extra poles, known as inter poles, reversing poles, or commutating poles, are provided midway between the main poles of the machine. Each is, of course, of the polarity of the pole which the armature is approaching. (*See* Fig. 99.)

As we saw in (*a*) the strength of field required is proportional to the armature current, and this is obtained by making the inter poles carry a current which is a definite fraction of the armature current; they are usually in series with the armature. A combination of this method, together with the use of

carbon brushes, is most general to-day for large machines. No lead, or practically no lead, is then, of course, required.

§ 5. **Calculation of Reactance Voltage.**—In large machines with bar conductors it is possible to have only one loop per commutator segment.

In small machines, however, where the segments cannot be made sufficiently thin for this, or in coil windings, there may be several loops to each commutator segment. It may readily be seen that the

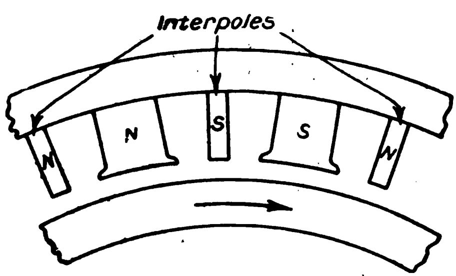

FIG. 99.—DIAGRAM OF MACHINE WITH INTER-POLES

reactance voltage varies with the square of the square of the number of loops. If there are x loops, the flux produced will be x times as great. The reactance voltage depends on the number of flux linkages, and, since this flux is linked with each of the x loops, will therefore be x^2 times as great.

It will be seen from a consideration of Fig. 100 (a) and (b), that the short circuit of the coil or coils between commutator segments C and D lasts from the moment that the heel of the brush touches C till the toe leaves D or, in other words, while the commutator moves a distance equal to the bush thickness less the thickness of the mica insulation between commutator

segments. If β = the thickness of the brush and b thickness of mica insulation,

$$t = \text{(time of commutation)} = \frac{\beta - b}{2^{\pi}\ \rho^{n}} \text{ where}$$

ρ = radius of commutator, and
n = revs. per sec.

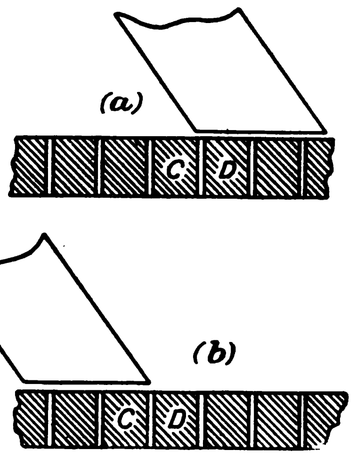

FIG. 100.—DURATION OF COMMUTATION

If L is the self-inductance of each loop (flux linkages for a current of 1 amp.) the reactance voltage = Lx^2di/dt where di/dt = rate of change of current.

Now the current changes from $+ i$ to $- i$ in time t, so that the average reactance E.M.F.

$$= \frac{2Lix^2}{t} \times 10^{-8} \text{ volts.}$$

Hobart has investigated the values of L, and he divides the loop into those portions embedded in the armature core and those portions which are free in air. He found that 1 amp. produces a flux of 4 C.G.S. lines

for each cm. of embedded length, and ·8 C.G.S. line for each cm. of free length.

Considering the embedded portion of the top conductor, the bottom conductor of the same slot is producing the same number of lines, and is also undergoing commutation at the adjacent brush. The same applies to the embedded length of the lowest conductor of the loop. Hence, the flux linkages and changes of flux linkages with the embedded portions are doubled.

$$\therefore \quad L = 8l_e + \cdot 8l_f$$

where l_e is the embedded and l_f the free length.

The reactance voltage, therefore,

$$= \frac{2Lix^2}{t} \times 10^{-8}$$

$$= \frac{16(l_e + \cdot 1l_f)x^2i}{t} \times 10^{-8} \text{ volts.}$$

(l_f is usually about three times pole pitch).

If the brush is so wide as to cover more than 2 commutator segments so that the m loops, or sets of loops, are being simultaneously short circuited, the self produced flux linkages will be approximately m times as great, since the loops very nearly completely overlap; but the time during which commutation takes place in each loop is now, of course, increased. The reactance voltage will therefore

$$= \frac{16\ (l_e + \cdot 1\ l_f)\ x^2mi}{t} \times 10^{-8} \text{ volts.}$$

Hobart lays down that this value should not exceed about 2 volts.

§ 6. **Armature Reaction.**—The ampere conductors on the armature, covering, as they do, the iron core, produce polarity in the armature which has an effect on the main field.

Consider the 4 pole machine in Fig. 101. (The reasoning may equally well be applied to a machine with any number of poles.) With a clockwise direction of rotation, the currents induced in the various conductors will be as shown. Dividing these into groups by the broken lines through the centre of the armature, we see that the effect is to produce polarity in the armature as indicated by the small letters *n* and *s*.

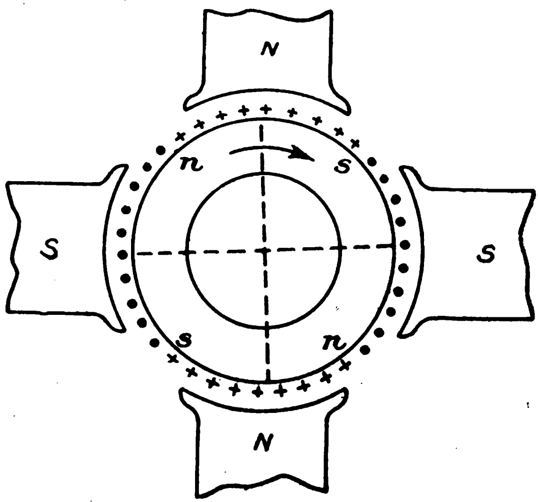

FIG. 101.—PRODUCTION OF POLARITY IN ARMATURE

The flux from the N. poles of the field, instead of ceasing to enter the armature at the points midway between the poles, is now encouraged further round in a clockwise direction. The points of zero entry of flux are now no longer midway between the poles, but at points nearer to the poles which the armature is approaching. In other words, the flux is sheared round, or the machine is said to be cross magnetized

(*see* Fig. 102). The current induced will, therefore, no longer change direction at the points midway between the poles, but at the new points of zero flux entry as shown in Fig. 103.

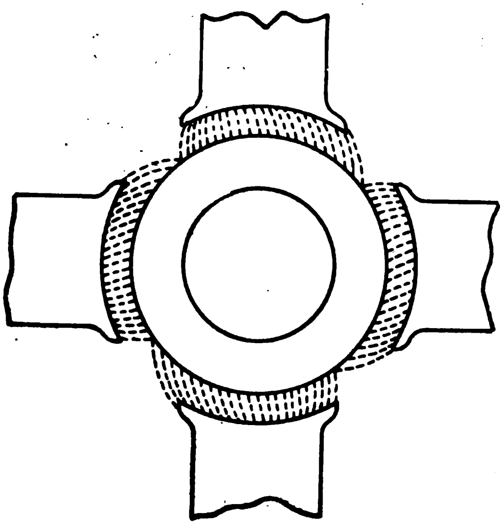

FIG. 102.—CROSS MAGNETIZATION DUE TO ARMATURE POLARITY

The brushes must therefore be placed at these points (*i.e.*, given a lead). It should be noted that this reason for giving the brushes a lead is different from that described in § 4. If commutating poles are used, they will tend to overcome this cross magnetizing effect in addition to producing the commutating effect, so that the brushes will then require no lead.

7. Demagnetization Produced by Brush Lead.—It will be realized that the distribution of currents in the armature conductors having been disturbed, the

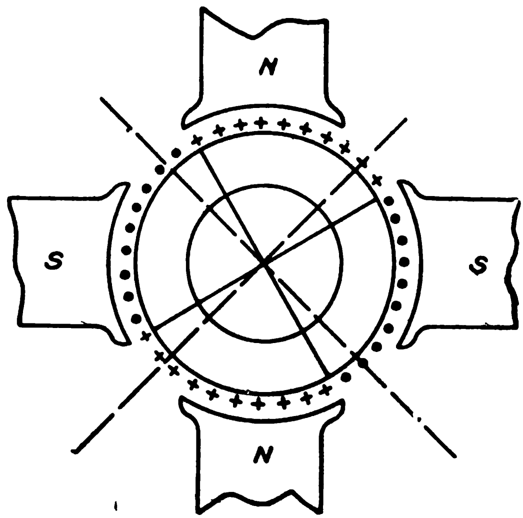

FIG. 103.—BRUSH "LEAD" DUE TO CROSS MAGNETIZATION

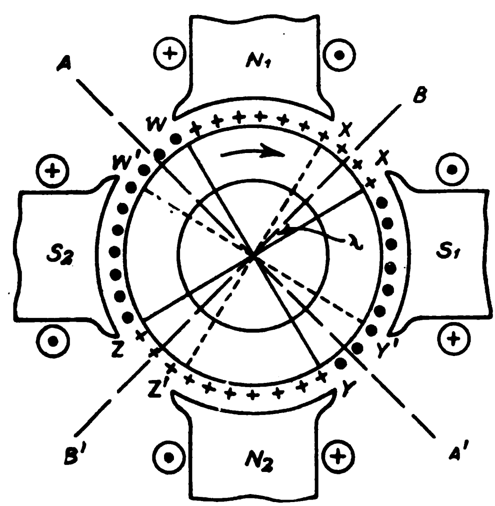

FIG. 104.—DEMAGNETIZATION DUE TO BRUSH "LEAD"

directions of flow are no longer symmetrically situated with regard to the poles.

In Fig. 104 the brushes are placed at $W\ X\ Y\ Z$, such that the angle of lead

$$WOA = XOB = YOA' = ZOB' = \lambda.$$

If $W'Y'$ and $X'Z'$ be drawn, making an angle λ with AA' and BB' respectively, but on the opposite sides from WY and XZ, we see that the belts of conductors WX', XY', YZ', and ZW' are symmetrically placed under the poles. Considering the pole N, however, we see that the conductors between $W'W$ and $X'X$ tend to produce a field in the opposite direction from the field winding on N_1. A similar state of affairs is occurring at the other poles.

Thus, the brush lead causes a tendency to demagnetize the field poles equal in ampere turns to the armature current multiplied by the number of conductors contained within an angle 2λ.

Thus, if there are N conductors on the armature, I_a is the current in each conductor and λ is the angle of brush lead, the ampere turns tending to demagnetize each pole

$$= I_a \times \frac{2\lambda}{2\pi} \times N$$

$$= \frac{\lambda I_a N}{\pi}.$$

This quantity was not considered in Chapter VI, and an equal number of ampere turns must be placed on the poles in addition to those calculated before, in order to overcome the effect.

Examples

(1) In a certain machine the commutation of each loop takes place in $\frac{1}{2300}$ sec. The embedded length

of conductor is 10 ins., and the pole pitch is 29 ins. Find the reactance voltage per loop if the current in each armature conductor is 30 amps.

(2) Define "reactance voltage" of D.C. machines. Calculate its value for a machine in which—

Net length of armature	= 12·5 cms.
Length of mean turn	= 100 cms.
Current per section	= 100 amps.
Loops per segment	= 1
Width of brush	= 3 segments.
Frequency of commutation	= 500 per sec.

[*Lond. Univ. Elec. Mach.*, 1912.]

(3) What is cross magnetization, and why does this necessitate brush lead? What is the effect of brush lead on the field? A 6 pole lap wound machine is delivering a current of 240 amps. There are 360 conductors on the armature and the lead to the brushes is 15 per cent. of the polar pitch. Find the demagnetizing ampere turns.

CHAPTER IX

LOSSES

§ **1.** Loss of power in a dynamo may be divided under three headings—

(1) Copper losses.
(2) Iron losses.
(3) Mechanical losses.

The first includes the power wasted in the field and armature windings, and may be calculated with considerable accuracy. The second includes the hysteresis and eddy current loss, which occurs mainly in the armature. The third includes the friction of the brushes and bearings, to which must be added the loss due to " windage " on air friction. There is also an additional loss which has not been included in the above. This is the loss due to resistance at the brush contacts. This may be included in the copper losses, but, as will be seen later, it is not scientifically accurate to do so, although it may be done in cases where only approximate results are required.

§ **2. Efficiency.**—The efficiency of any machine is the ratio of energy supplied to energy delivered. It is obvious that this ratio must always be less than unity. Suppose that an engine whose brake horse-power is H is supplying energy to a dynamo which is delivering a current I at a voltage E. Then the efficiency, usually denoted by η, is given by

$$\eta = \frac{\text{output}}{\text{intake}} = \frac{E \times I}{H \times 746}.$$

The difference between $H \times 746$ and EI is the total number of watts lost in the generator.

This value of η is known as the Commercial or Gross efficiency.

Efficiency may also be written

$$\eta = \frac{\text{Electrical output}}{\text{Electrical output} + \text{losses}}.$$

In gross or commercial efficiency, losses of all kinds are taken into consideration, but if we consider the electrical losses and neglect the mechanical losses, we have what is known as the electrical efficiency, which may be written

$$\frac{EI}{EI + (I^2R \text{ losses})}$$

from which it is obvious the efficiency varies with the load, so that a machine is designed to have its maximum efficiency when delivering its full load. When a machine is being designed, its probable efficiency may be calculated with considerable accuracy, since we have empirical formulae which allow us to estimate the various losses.

§ 3. **Copper Losses.**—These losses may be calculated since we have the resistances of the various windings. Thus, if I_a is the armature current and R_a armature resistance, I_f the current through the field coils of resistance R_f, then the total copper loss is

$$I_a^2R_a + I^2{}_fR_f$$

It must be noted that these resistances vary with their temperature, therefore R_a and R_f are the resistances when the machine is running at its maximum temperature (*see* later). The copper loss in the armature is usually about 45 per cent., and in the field about 25 per cent., of the total losses in a well-constructed machine.

§ 4. **Iron Losses.**—These may in turn be divided under two heads—

(*a*) Hysteresis loss.

(*b*) Eddy current loss.

(*a*) Hysteresis Loss.—In order to calculate this we may use Steinmetz' Empirical Law.

Loss in ergs per cc. per cycle $= \eta B^{1 \cdot 6}_{\text{max.}}$ Here again we encounter a difficulty in the fact that the value of the induction B varies considerably, being much greater in the teeth than in the core of the armature, therefore we must consider these portions separately.

(i) *Loss in the Teeth.* Since the slots are rectangular the teeth taper downwards, hence the flux density varies over their depth. In order to calculate the loss we must take an average value for the flux density, which we assume to occur half-way down the slot.

The value of the induction in the teeth is given by

$$B = \frac{\text{Flux per pole}}{\text{Sectional area of the teeth under pole}}$$

Thus, if Φ is the flux per pole, N the number of teeth under the pole, l their length, and x their average width (ab) (Fig. 105) the average flux density is given by

$$B = \frac{\Phi}{N \,.\, l \,.\, x}$$

It is assumed in the above that the whole of the flux passes through the teeth. This is only approximately true, but sufficiently accurate for practical purposes. Further, since the induction in the teeth is very high, Steinmetz' Law no longer holds, but gives values which are too high. So to calculate the loss we use tables which have been compiled, giving the hysteresis loss corresponding to these high values of

B. The following table gives a series of values for soft wrought iron—

Induction (*B*), C.G.S. units	17000	18000	19000	20000	21000	22000	23000	24000
Ergs per C.C.M. per *CYCLE* . . .	20000	23000	25000	27000	28500	29000	29700	30000

(ii) *Core.* Here again induction varies from point to point, but knowing the shape of the magnetic circuit we may arrive at an approximate value of the average induction throughout the core and calculate the loss either from tables as in (i) or by Steinmetz' Law.

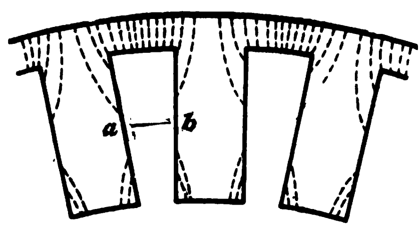

FIG. 105.—VARIATION OF FLUX DENSITY IN ARMATURE TEETH

§ 5. **Eddy Current Loss.**—This must be divided into two portions, as in the case of hysteresis loss.

(i) TEETH.—It has been shown in § 3, Chap. VII, that the eddy current loss varies inversely as the square of the number of laminations. Further, it is also proportional to square of the number of revolutions per second. The E.M.F. induced, and therefore the current produced by it is proportional to the speed, and since the eddy current loss is proportional to the square of the current, it is therefore proportional to the square of the number of magnetic reversals per second. Similarly, it is proportional to the square

of the induction B. Therefore, the eddy current loss in the teeth may be written

$$K\,(B \cdot \eta \cdot T)^2 \text{ watts per sec.}$$

When T is the thickness of the laminations in cms.

§ 6. The above calculations are only approximate, since many assumptions have been made which are not strictly true. Some of these have already been indicated. The errors which arise in the eddy current calculations are caused by assuming that the induction

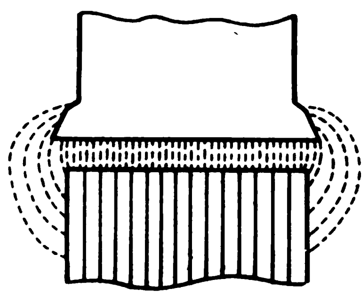

FIG. 106.—FRINGING AT THE POLE TIPS

is always in planes parallel to the laminations. This is evidently not justified, since the flux at the edges of the pole piece may be almost at right angles to the laminations, and therefore cause a considerable increase in the loss due to eddy currents. Further, all the laminations may not be completely insulated from each other, since the armature is turned after the stampings have been fitted and some of the edges may become burred. We have also neglected the eddy current loss occurring in the bolts and conductors of the armature, and, if bar conductors are used, the eddy currents in these may be comparatively high.

§ 7. **Mechanical Losses.**—These may be classified as follows—

(*a*) Brush friction.
(*b*) Bearing Friction.
(*c*) Windage.

(*a*) Brush Friction.—This depends on the pressure per unit area of brush surface in contact with the commutator, the coefficient of friction for the two surfaces, and the peripheral speed of the commutator.

If μ is the coefficient of friction, v the peripheral velocity in ft. sec., P pressure per square inch, in lbs., and A the area in contact. Then the loss due to brush friction is given by

$$\frac{\mu PAV \times 746}{33000} \text{ watts}$$

The coefficient of friction between carbon and copper has an average value of about ·3. This may be reduced to ·05 by lubricating the surface of the commutator with a small quantity of paraffin wax, but the effect soon disappears.

(*b*) Bearing Friction. This loss will vary with the surface in contact and the peripheral speed of the shaft, and may be calculated from the following formula

$$\text{Watts lost} = dlv^{1\cdot5} \times 10^{-3}$$

Where d is the diameter and l the length of the surfaces in contact (in inches), and v is the velocity in feet per minute.

(*c*) Windage. This loss can only be obtained approximately. It depends to a considerable extent on the construction of the machine, and is found to increase very rapidly with the speed, and in high speed machines assumes considerable importance.

For a machine of about 20 kws. output, the friction and windage losses are of the order of 2 per cent.

§ 8. **Loss Due to Contact Resistance.**—In § 1 we stated that it would be an advantage not to include the loss due to brush contact resistance with the armature copper loss. The loss here is given by I^2R, where R is the contact resistance and I the current through it. This resistance, however, is not constant, it varies both with the current density and the temperature. Further, the losses at the negative and positive brushes do not vary in the same manner. The resistance of a brush is found to vary inversely

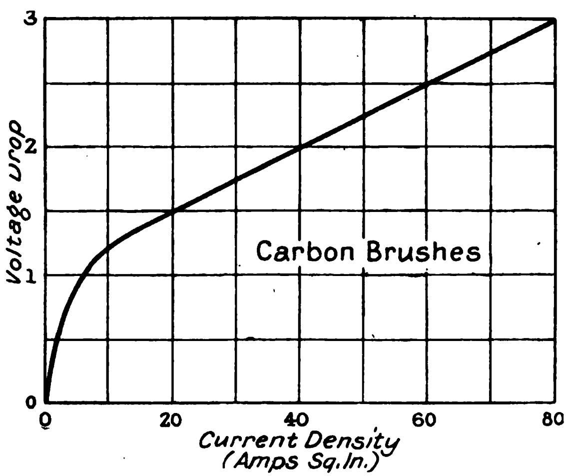

FIG. 107.—VOLTAGE DROP AT BRUSH CONTACTS

with the current; that is to say, when the current increases the resistance decreases, and therefore the voltage drop at the brushes does not increase at a rate proportional to the current.

The curve shown in Fig. 107 has been determined from extensive experiments, and shows the variation

of the average voltage drop with current density for both positive and negative brushes.

The current density is given by dividing the armature current by half the total area of brush contact. To obtain the loss due to this drop in voltage, the armature current is multiplied by the corresponding voltage drop read off the curve.

§ 9. Temperature Rise in Armature.—The whole of the energy wasted in a machine is dissipated in the form of heat and the consequent rise in temperature limits the maximum output to a considerable degree. Thus, if the temperature of the armature or field windings rises above about 70° C., the insulation of the conductors is endangered.

This rise in temperature depends on

(i) The rate at which energy is lost.

(ii) The cooling surface and general design of the machine.

Let t represent the rise in temperature.

Then $$t = K\frac{W}{A} \qquad \qquad (12)$$

Where W represents the watts lost and A the total external surface of the machine, K is a constant and depends upon the design; it will vary with the type of radiating surface and the speed, and will not be the same for an enclosed as for an open type machine.

Equation (12) may be written—

Rise in temperature in °C. = K (watts lost per unit area of radiating surface)

Values of K are given in Fowler's *Electrical Engineer's Pocket Book.*

The temperature rise may be determined experimentally by running the machine continuously at full

load, stopping it every half hour and measuring the temperature until the reading becomes constant or nearly so. The temperature-time curve will be nearly exponential in form. The slight variation arises from the fact that the power lost increases somewhat with the rise in temperature. For typical curve, *see* Fig. 108. In small machines the time taken to reach maximum temperature is about 4 hours, medium machines 6 hours, and large machines 8–10 hours.

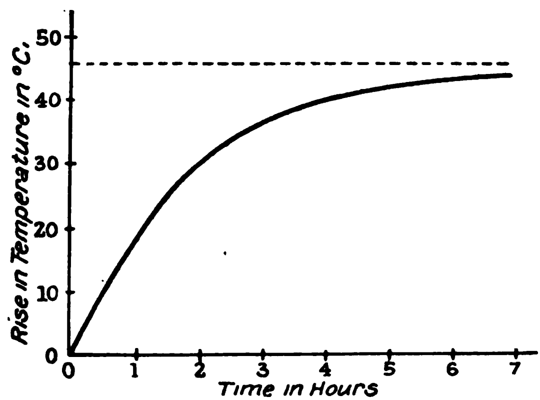

FIG. 108.—TEMPERATURE-TIME CURVE

§ 10. **Experimental Analysis of Losses.**—The various losses may be separated for a direct current dynamo or motor in an extremely simple manner. It has been shown that—

(i) The hysteresis loss is directly proportional to the speed, and

(ii) The eddy current loss is proportional to the square of the speed.

Therefore, the losses in the armature core may be written

$$W_a = An + Bn^2$$

where A and B are constants depending on the design of the machine and the field strength. n is the speed in revolutions per second.

The other losses in the armature are due to friction, and may be written in the form of a general expression in ascending powers of n.

$$\text{Friction losses} = Cn + Dn^2 + En^3 + \ldots\ldots$$

Since, however, the speed of the machine does not vary between wide limits, terms containing higher powers than n^2 may be neglected. C and D are constants which depend only on the machine and not on the strength of the field. The total losses, therefore, may be written

$$W = An + Bn^2 + Cn + Dn^2$$

This may be re-written

$$\frac{W}{n} = A + C + (B + D)n\,.$$

This is the equation of a straight line where $\frac{W}{n}$ and n are the variables.

Suppose, then, that we run a machine as a motor (Fig. 109) which is separately excited (normal exciting current) and measure the voltage and armature current corresponding to various speeds. We may thus obtain the power required to run the armature at various speeds

$$W = EI - I^2R$$

where R is the armature resistance.

From these readings we may now plot a curve showing the relation between $\frac{W}{n}$ and n, which will be very approximately a straight line (EF in Fig. 110). This line will represent

$$\frac{W}{n} = A + C + (B + D)n \quad . \quad . \quad (13)$$

Hence, $OY = A + C$. . . . . . (14)

$\tan\theta = B + D$ . . . . . (15)

Y is the point where the line, when produced, cuts the Y axis, and θ is the inclination of the line to the X axis.

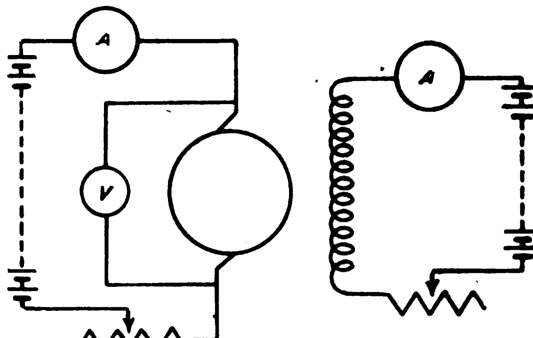

FIG. 109.—CONNECTIONS FOR EXPERIMENTAL ANALYSIS OF LOSSES

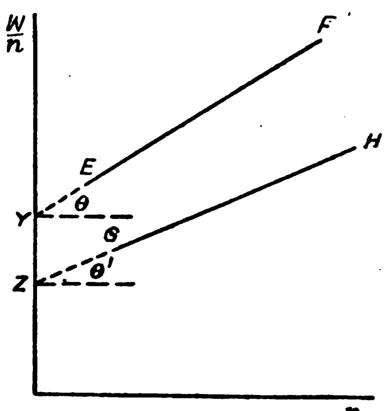

FIG. 110.—ANALYSIS OF ARMATURE LOSSES

We have now separated the two sets of losses proportional to the speed and to the square of the speed, respectively.

Suppose now that we reduce the excitation of the machine and repeat the experiment. A different line will be obtained, since the hysteresis and eddy current losses are now reduced but the friction and windage losses remain the same (i.e., A and B change, say, to A' and B', while C and D remain as before).

Let the new line be repeated by GH

then $A' + C = OZ$ . . . . . (16)

and $B' + D = \tan \theta'$ . . . . . (17)

From these equations we obtain

$$A - A' = ZY$$

$$B - B' = \tan \theta - \tan \theta'$$

The relation between hysteresis losses for different values of the induction are known (Steinmetz' Law), and in the case of eddy current loss we know that the loss varies as the square of the speed.

Therefore, if the total flux for normal excitation is Φ and for the reduced excitation is Φ'

$$\frac{A'}{A} = \left(\frac{\Phi'}{\Phi}\right)^{1.6} \text{ and } \frac{B'}{B} = \left(\frac{\Phi'}{\Phi}\right)^{2}$$

The value Φ'/Φ may be determined from the *total* E.M.F. generated by the machine, which may be calculated from the voltmeter reading and the armature drop ($R_a I$).

Let E' and E be these two values, then obviously

$$\frac{\Phi'}{\Phi} = \frac{E'}{E} = K.$$

Therefore, $A' = AK^{1.6}$ and $B = B'K^2$

Hence, $$A = \frac{YZ}{I - K^{1.6}} \text{ and } B = \frac{\tan \theta - \tan \theta'}{I - K^2}$$

Knowing A and B, the losses at normal excitation for any speed $\bar{n}$ may now be calculated.

(1) Hysteresis loss $= A\bar{n}$

(2) Eddy current loss $= Bn^2$

(3) Friction, etc., losses $=$ Total loss (from curve) $- [A\bar{n} + B\bar{n}^2]$

§ 11. **Running Down Method.**—Suppose that a normally excited machine is run up to a little over its normal speed, and then the driving power is suddenly removed. The machine will slow down more or less quickly, according to the magnitude of the resisting forces. Therefore, if we can measure the rate of slowing down when the speed is normal, we shall have a measure of the losses.

Let K represent the amount of the inertia of the armature.

Its kinetic energy when revolving at a speed n is

$$2K\pi^2n^2$$

and, therefore, the rate of loss of energy at any instant during slowing down is

$$\frac{d}{dt}(2K\pi^2n^2) = 4K\pi^2n\frac{dn}{dt}$$

Therefore, the core losses at normal speed are given by

$$\left(4K\pi^2n\frac{dn}{dt}\right)_{n\,=\,\text{normal speed}}$$

This only involves two unknowns: (1) The moment of inertia of the armature and flywheel, and (2) the rate of slowing down at normal speed. If, therefore, we run the armature up to a little over its normal speed, cut off the power and take simultaneous readings of time and speed, a slowing down curve (Fig. 111)

may be plotted. The slope of the curve at the point corresponding to normal speed will give $\left(\frac{dn}{dt}\right)_{n=N}$ and the subnormal AB at that point will give $\left(n\frac{dn}{dt}\right)_{n=N}$ where N represents the normal speed.

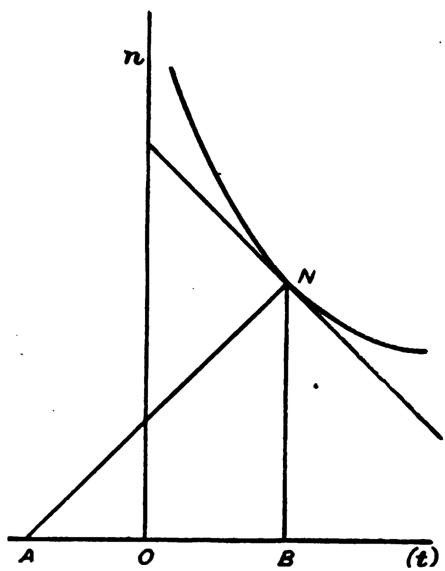

FIG. 111.—RUNNING DOWN CURVE.

If the above experiment is carried out with the normal exciting current flowing in the field coils, the power wasted will be due to the whole of the core losses. If, now, we obtain a second curve with the field unexcited, we shall have a curve showing the effect of the frictional losses only.

From these curves, then, we may calculate the values of (1) Frictional, (2) Hysteresis and Eddy

current losses corresponding to any speed. Further, the latter may be separated as detailed in the preceding paragraph, since the hysteresis loss is proportional to the speed, and the eddy current loss to the square of the speed.

The moment of inertia of the flywheel may be calculated from the dimensions of armature and flywheel, or found experimentally in a number of ways. An obvious method is as follows. Attach a disc of known moment of inertia (K') to the flywheel and determine another slowing down curve. The rate of slowing down will not be the same. Let the new rates be $\frac{dn'}{dt}$ when $n = N$. The armature loss as before, is given by

$$4\pi^2(K + K')\left(n\frac{dn'}{dt}\right)_{n=N}$$

and since the losses in the two cases are equal (neglecting a slight increase in friction and windage loss), we have

$$K = K'\frac{\frac{dn'}{dt}}{\frac{dn}{dt} - \frac{dn'}{dt}}$$

Examples

1. A series dynamo supplies a current of 11 amps. at 30 volts. The armature resistance is ·35 ohm. and the resistance of the field coils is ·6 ohm. Find the electrical efficiency of the machine.

2. A shunt wound dynamo supplies a current of 58 amps. at 60 volts. The friction and windage losses are 3 per cent. of the output. The armature resistance is ·06 ohm, the field resistance 14 ohms. Calculate the commercial efficiency of the machine.

(3) The following are data of the armature of a 6 pole machine running at a speed of 800 revs. per min.:

Length of armature	=	16 cms.
Depth of slot	=	1·7 cms.
Average width of tooth	=	·8 cms.
Number of teeth	=	100
Induction in teeth	=	16000
Average induction in core	=	9000
Total volume of core	=	$1{\cdot}2 \times 10$ ccs.

Assuming $\eta = {\cdot}003$, calculate the loss due to hysteresis: (*a*) in the teeth, (*b*) in the armature core.

CHAPTER X

OPERATION OF THE DYNAMO

§ **1.** In Chapter VII four types of field windings were discussed: separate, series, shunt, and compound; we now proceed to treat each type in turn and to examine the effect of these various methods of excitation on the operation of a machine.

A dynamo is usually designed to run at a definite constant speed; therefore, if we can deduce a relation between the terminal voltage and the current supplied when running at this speed, we shall have some idea of the manner in which the machine operates and of its practical limitations. A curve showing a relation of this type is known as a characteristic.

§ **2. Separately Excited Dynamo.**—Here we assume that the dynamo is driven at constant speed and that the current in the field coils is constant. In an ideal machine the E.M.F. generated would be constant, and the characteristic curve showing the relation between terminal volts and current would be a straight line BA (Fig. 112). In the actual machine, however, this would not be the case.

It has been shown in Chapter VIII that when a current is flowing in the armature, a demagnetizing flux is created, so that the effective flux is always less than the actual flux due to the field magnets, the diminution in flux being due to the demagnetizing ampere turns on the armature. This effect of armature reaction increases as the load current increases, therefore the characteristic curve will not be represented by BA but by a line falling away from B to C.

We have still another effect to take into consideration. When a current I is flowing in the armature,

resistance R, the voltage drop in the armature is given by RI, so that in a dynamo generating a voltage E, the actual terminal voltage will always be less than E by an amount RI. This voltage drop due to armature resistance may be represented by a straight line (OD) through the origin, and is known as the *internal characteristic* of the machine.

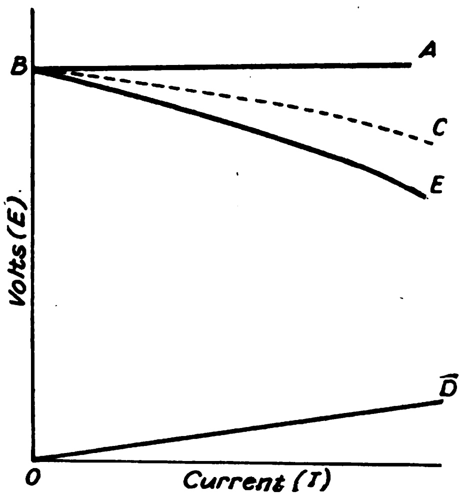

FIG. 112.—CHARACTERISTICS OF A SEPARATELY EXCITED DYNAMO

Subtracting from the voltage given by points on the line BC the corresponding drop due to armature resistance, we obtain a curve BE, which will give the terminal voltage corresponding to any load. BE is known as the *external characteristic*, and BC as the total characteristic, since the latter represents the total E.M.F. generated by the machine.

§ **3. Series Dynamo.**—In this case the exciting current is the same as the line current, and therefore

the flux and E.M.F. increase with the load (up to a certain point). It is necessary first to consider the conditions under which a series machine will excite its own field. It is obvious that there must be some residual magnetism in the field magnets, and that the field coils must be so connected to the armature that the current which is generated tends to increase the flux and not to diminish it. Further, if the resistance in the external circuit is too great, the exciting current will be so small that there will be no appreciable increase of flux, and therefore the machine will not excite itself. The resistance at which the machine just excites is known as the *Critical Resistance.*

§ **4.** Let Φ_o represent the residual flux through the armature, and suppose that as a result of this flux the E.M.F. generated produces a very small current I (amps.). This current flowing in the field winding excites a flux, say, $\Phi_1 I$. It is assumed that the flux is proportional to the current, since the latter is extremely small. This assumption is approximately true, since the magnetizing force is small, and in the initial portion of the BH curve, B is roughly, proportional to H.

∴ The total flux now passing through the armature is given by

$$\Phi = \Phi_o + \Phi_1 I$$

∴ The E.M.F. generated is

$$E = [\Phi_o + \Phi_1 I] \; Nn10^{-8} \text{ volts} \quad . \quad (18)$$

where $N =$ number of conductors

and $n =$ number of revs. per sec.

Let R represent the total resistance of the circuit in armature field coils and external circuit.

Then $E = RI$ . . . . . . (19)

Equating (18) and (19), and re-arranging, we have

$$[R - \Phi_1 Nn10^{-8}]I = \Phi_0 Nn10^{-8}$$

and $$I = \frac{\Phi_0 Nn10^{-8}}{R - \Phi_1 Nn10^{-8}}$$

Now, since Φ_0 is very small, the numerator of the fraction is very small. Therefore, for a finite current, the denominator must approximate to zero, or

$$R = \Phi_1 Nn10^{-8} \quad . \quad . \quad . \quad . \quad (20)$$

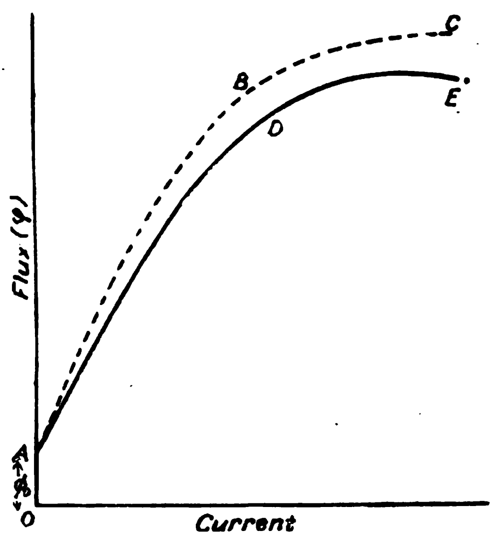

FIG. 113.—MAGNETIC CHARACTERISTICS OF THE SERIES DYNAMO

§ 5. **Characteristic Curves of the Series Dynamo.**—(1) MAGNETIC CHARACTERISTIC.—This is a curve showing the relation between the flux through the armature and the exciting current, which in this case is the load current.

At no load we have a small flux Φ_0 due to the residual magnetism.

If armature reaction were absent, *ABC* would represent the general shape of the magnetic characteristic, but owing to the demagnetizing effect of the armature current, the useful flux is always less than the flux due to the field coils, and since the demagnetizing effect increases with the load, the actual magnetic characteristic *ADE* drops away from *ABC* at high values of the load current.

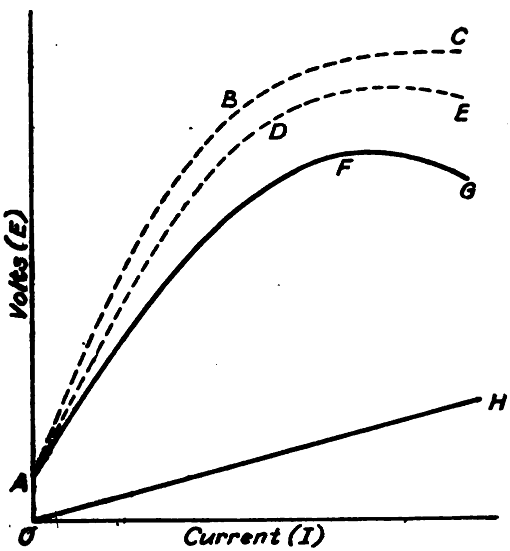

FIG. 114.—CHARACTERISTICS OF THE SERIES DYNAMO

§ 6. E.M.F. Characteristic.—This may be obtained directly from the magnetic characteristic *ADE*, and since the E.M.F. generated is proportional to the flux, the curve will have the same shape. Curve *ABC* (Fig. 114) will represent the case, assuming no

armature reaction, and ADE will be the total characteristic allowing for the voltage drop due to armature reaction.

As before, we must deduct the drop in voltage due to armature resistance, but in this case we must also deduct the volts lost in overcoming the resistance of the series winding. These two losses are represented

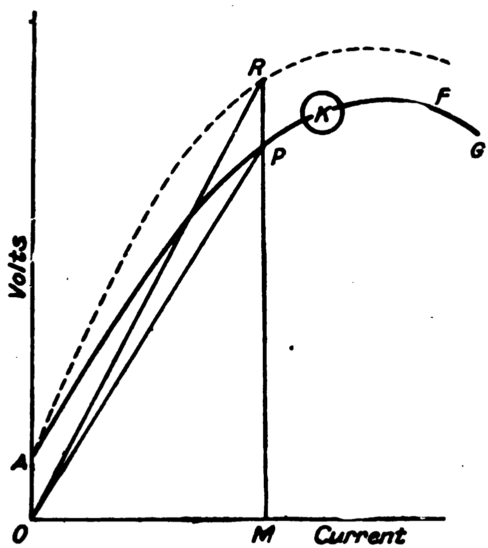

FIG. 115.—RESISTANCE CHARACTERISTIC OF THE SERIES DYNAMO

by the internal characteristic passing through the origin. The external characteristic may therefore be represented by a curve whose shape is similar to AFG.

§ 7. **Resistance in External Circuit.** — Let AFG (Fig. 115) represent the external characteristic. Draw any line OP through the origin, cutting the external

characteristic at P. Let $(OM: PM)$ be the co-ordinates of point P.

Then $\dfrac{PM}{OM} = \dfrac{\text{E.M.F.}}{\text{current}} =$ resistance in the external circuit.

Similarly, a line OR cutting the total characteristic at R gives the total resistance (armature + external) in circuit; *i.e.*,

$$\frac{RM}{OM} = \frac{\text{E.M.F.}}{\text{current}} = \text{total resistance.}$$

From these curves we may obtain an approximate value of the critical resistance. Let the angle POM be represented by θ. Suppose that θ is increased until the line OP is approximately a tangent to the characteristic curve. The line OP can never be a tangent to the curve, but, since the latter commences at a point very close to the origin and its initial portion is straight, OP may be arranged so that it lies roughly along the curve. Now

$$\tan\theta = \frac{PM}{OM} = \frac{\text{E.M.F.}}{\text{current}} = R$$

where R is the external resistance.

So in this region of the curve, if the value of $\tan\theta$ and therefore of R is changed slightly, large variations in E.M.F. and current will be caused. Hence, over this range the machine is very unstable, so that the usual working portion of the characteristic is about some point beyond, say, at K, where the conditions are more stable. Further, this value of θ, when OP is approximately a tangent to the curve, gives the critical resistance of the machine, for, if θ, and therefore R, is increased beyond this limiting value, it is obvious that the machine will not have the conditions necessary for self excitation.

§ **8.** Suppose that we take a series of points along

the curve APG, and from the values of the co-ordinates E and I calculate the corresponding values of the resistance R. If next we plot a curve connecting the volts E with this resistance (R) (*i.e.*, of the external

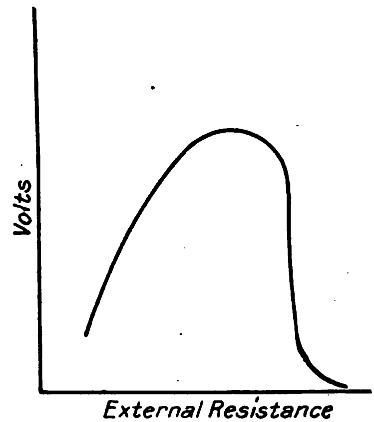

FIG. 116.—CURVE SHOWING THE RELATION BETWEEN THE TERMINAL VOLTAGE AND EXTERNAL RESISTANCE FOR A SERIES DYNAMO

circuit), we shall obtain a curve whose shape will be similar to that shown in Fig. 116. The sudden drop in the volts at a given value of R shows the value of the critical resistance quite distinctly.

§ 9. **Power Lines.**—Returning to the external characteristic, we may draw on the same scale a series of curves known as power lines. The output in horsepower when the terminal voltage is E and the load current is I amps., is given by

$$\frac{EI}{746}$$

If, then, we plot a series of curves so that the product of the co-ordinates divided by 746 is equal to

1, 2, 3, 4, etc., we shall have a series of rectangular hyperbolae with the asymptoles as axes representing lines of equal output cutting the characteristic. These lines, of course, are the same for any type of machine, series, shunt, or compound.

§ 10. **Uses of Series Wound Generators.**—Since most distribution systems for power and lighting require a constant voltage, it is clear that a series generator is

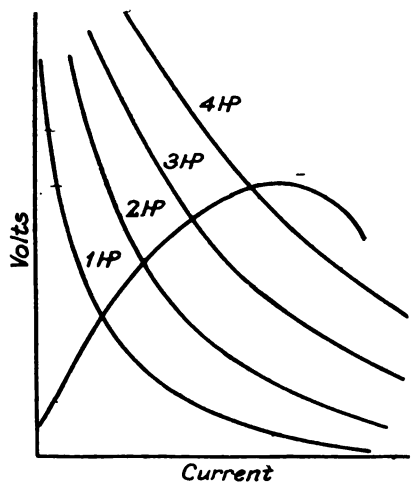

FIG. 117.—POWER LINES

not suitable. Further, it is important to note that a series machine must not be used for charging accumulators, since, if the battery E.M.F. becomes greater than the E.M.F. generated by the machine, the current will be reversed in both armature and field coils, therefore the machine will run as a motor in the same direction. The resistance of armature and field coils is very small, and therefore the current through them will become dangerously high.

Series generators are used in a special system of series arc lighting. Here, a constant current is required at voltages which are proportional to the load.*

The machines are designed so that the external characteristic has a pronounced droop.

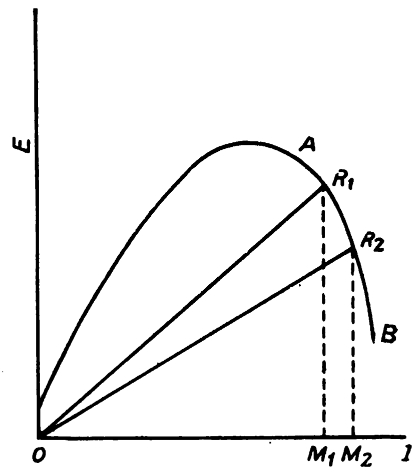

FIG. 118.—CHARACTERISTIC OF A SERIES DYNAMO USED FOR SERIES ARC LIGHTING

This would be equivalent to the machine having a large armature reaction beyond a certain load. This may be arranged when designing the machine, and the droop is also accentuated by employing a special regulator which shunts some of the field coils and so produces a variation in voltage while the current is approximately constant.

The working portion of the curve is the region denoted by AB (Fig. 118). It will be seen from the diagram that over this range the current will be,

* If the arcs are in parallel a fixed voltage and varying current is required.

roughly, constant while the voltage is proportional to the load; *e.g.*, take two loads denoted by tan θ_1 and tan θ_2,

$$\theta_1 = R_1\hat{O}M_1 \quad \theta_2 = R_2\hat{O}M_2.$$

The voltages are given by $R_1 M_1$ and $R_2 M_2$, which are proportional to tan θ_1 and tan θ_2 respectively, since OM_1 is sensibly equal to OM_2. If, therefore, one of the arcs in a series is "struck," a big rush of current will not take place, but only an inappreciable increase.

§ **11. Shunt Wound Dynamo.**—Imagine that the machine is ideal, that is, without armature reaction or resistance. The field windings are connected directly across the brushes, therefore when the machine is on open circuit the full E.M.F. is generated. Hence, the E.M.F. characteristic for an ideal machine would be a straight line parallel in the horizontal axis, but, as in the case of the two previous types, the effect of armature reaction and resistance is to cause the curve to droop so that the voltage falls away as the load increases. In addition to the drop due to these causes, there will be a further loss due to the reduction of the field current, because, when the load increases the voltage falls, therefore the field current is diminished, causing a diminution in flux, and therefore a decrease in the E.M.F. generated.

Suppose now that the resistance of the external circuit is decreased, the load current will not increase indefinitely, since a point will soon be reached when the resistance of the external circuit is so small that there will be a considerable diminution of the field current, and therefore a consequent drop in E.M.F. generated. The external characteristic, then, must droop downwards and finally pass through the origin. This follows since when the E.M.F. is zero the current is zero.

Thus, when the machine is short circuited there is no great rush of current. It is, however, impossible to take a machine through the complete curve because the current would be too great long before the theoretical maximum load is reached.

We have, then, for the shunt dynamo a critical resistance, but in this case, in order that the dynamo may function, the working resistance of the external

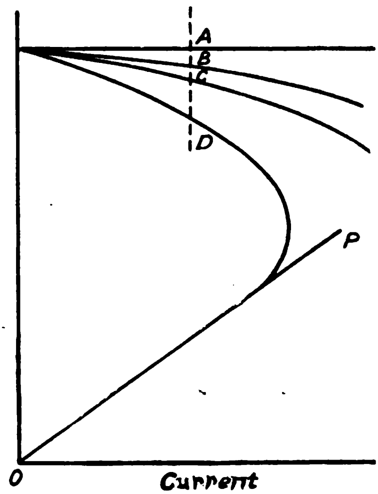

FIG. 119.—CHARACTERISTICS OF THE SHUNT DYNAMO

AB. Voltage drop due to armature reaction
BC. Voltage drop due to armature resistance
CD. Voltage drop due to decrease in exciting current

circuit must be greater than the critical resistance. In the series dynamo the working resistance of the external circuit must be less than the critical resistance.

As before, the critical resistance is obtained by drawing a line through the origin so that it is approximately a tangent to the characteristic. (*See OP*, Fig. 119.)

The critical resistance of the external circuit may be examined analytically as follows—

Let Φ_o represent residual flux and Φ_1 the flux through the armature due to a current of 1 amp. through the field coils. Let I_F represent the field current. Then the flux through the armature produced by this current is $\Phi_1 I_F$.

$\therefore$ The total flux (Φ) through the armature is given by

$$\Phi = \Phi_o + \Phi_1 I_F$$

$\therefore$ The total E.M.F. generated is

$$E = (\Phi_o + \Phi_1 I_F)\, Nn10^{-8} \text{ volts} \quad . \quad . \quad (21)$$

where N and n have their usual significance.

The voltage across the terminals of the machine E_T will be given by

$$E_T = E - R_a I \quad . \quad . \quad . \quad . \quad (22)$$

where R_a is the armature resistance and I the current through the armature. Let R_e represent the resistance in the external circuit, then neglecting the current I_F taken by the field coils in comparison with the load current,

$$I = \frac{E}{R_a + R_e} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Substituting this value of I in Equation (22) we have

$$E_T = E\frac{R_e}{R_a + R_e}$$

$\therefore$ The current in the field coils I_F is given by

$$I_F = \frac{E_T}{R_F} = E\frac{R_e}{R_F(R_a + R_e)} \quad . \quad . \quad (24)$$

Substituting this value of I in Equation (21)

$$E = \left\{\Phi_o + \Phi_1 E\frac{R_e}{R_F(R_a + R_e}\right\} Nn10^{-8} \quad (25)$$

Re-arranging, we have

$$E = \frac{\Phi_o N n 10^{-8}}{1 - \Phi_1 N n 10^{-8}} \left[\frac{R_e}{R_F(R_a + R_e)} \right] \quad . \quad (26)$$

Now Φ_o is very small, therefore the numerator of this fraction is very small. Hence, if a finite voltage

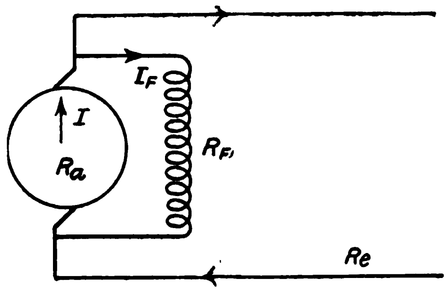

FIG. 120

is to be generated by the machine the denominator must approximate to zero, *i.e.*,

$$I - \Phi_1 N n 10^{-8} \frac{R_e}{R_F(R_a + R_e)} = O$$

$$\text{or} \quad R_e = \frac{R_F R_a}{\Phi_1 N n 10^{-8} - R_F}$$

For a finite E.M.F. to be generated, R must not be less than this value, which is known as the critical resistance of the external circuit.

Next, consider the machine on open circuit. In this case the armature and field coils may be considered to be in series. It follows at once that the field coils must not have a resistance greater than a certain critical value for the machine to be capable of self excitation. The analytical treatment is exactly the

same as in the case of the series dynamo. The critical resistance of the shunt winding being given by

$$R_F]_{\text{critical}} = \Phi_1 N n 10^{-8}$$

§ 12. **Uses of Shunt Dynamos.**—The shunt excited dynamo is generally used for lighting purposes, since it maintains an approximately constant voltage. Unlike the series wound machine, it may be used to charge accumulators.

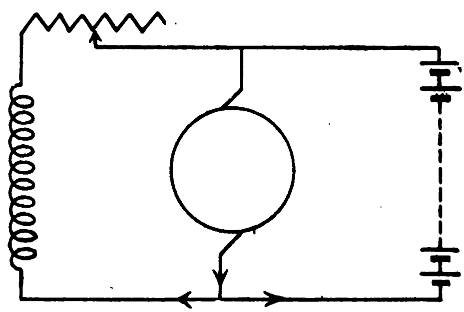

FIG. 121.—CONNECTIONS FOR A SHUNT DYNAMO CHARGING ACCUMULATORS

The machine is connected directly to the cells, and in the field, an adjustable resistance is inserted, so that the E.M.F. generated may be adjusted to the number of cells being charged. Suppose, for some reason, such as a momentary loss of speed, that the E.M.F. of the dynamo falls below that of the battery, the current will be reversed in the armature, but the current in the field coils will remain in the same direction, therefore the E.M.F. of the generator remains in the same direction, and there will be no large rush of current corresponding to a short circuit of the battery.

§ 13. **Compound Wound Dynamo.**—The characteristic of this machine may be almost any shape

depending on the degree of compounding. We have shown that the series machine gives a rising characteristic over the initial portion, while the shunt wound

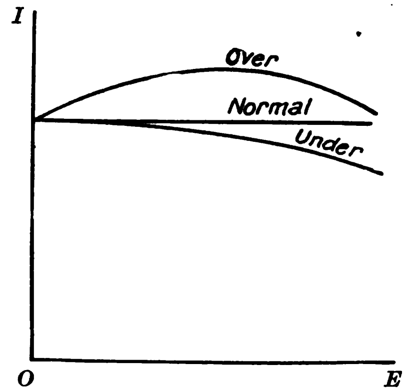

FIG. 122.—CHARACTERISTICS OF THE COMPOUND DYNAMO

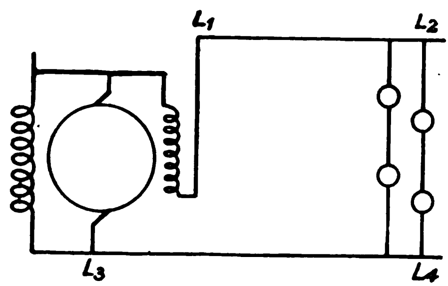

FIG. 123

type gives a drooping one. We may, therefore, divide compound machines into three types: over compounded, normal, and under compounded, according as the characteristic rises, is level, or falls. The type of field winding used depends upon the manner in

which the machine is to be used. Theoretically, we should be able to design a machine which would give a constant voltage for all loads (*i.e.*, a normal, compounded machine), but since the *BH* curve for iron flattens out at high values of *B*, the characteristic always droops slightly.

By over compounding a machine, the voltage delivered some distance away (say in a lighting system) may be kept constant.

In Fig. 123, suppose that L_1 L_2 and L_3 L_4 represent the feeders leading from a generating station. Let the no load voltage of the machine be represented by E_0. Let the resistances of L_1 L_2 and L_3 L_4 be each equal to R.

Then the voltage at L_2 L_4 is given by

$$E_0 - 2RI$$

where I is the load current. If, now, we can arrange the degree of over compounding so that the increase in E.M.F., due to the load current I flowing in the series coils, is equal to the voltage drop in the line (*i.e.*, $2RI$), we shall be able to deliver a voltage at L_2 L_4 which is independent of the load. This, of course, is not possible over anything but a small range.

CHAPTER XI

THE DIRECT CURRENT MOTOR

§ 1. Dynamo as Motor.—Since a wire carrying a current in a magnetic field is acted upon by a force normal to its length, a turning moment can be obtained upon the armature of a dynamo if, instead of rotating it by an outside agent, a current be passed in and out through the brushes. In fact, any dynamo may be used as a motor by applying from an external source a P.D. to the brushes.

In Fig. 124 is shown a series dynamo being rotated in a clockwise direction. Suppose the currents to be in the direction of the arrows. The electro magnetic pull on the conductors tends to oppose the motion giving rise to the current (Lenz' law), and is therefore in an anti-clockwise direction. If the armature be stationary and currents be passed in the direction of the arrows, the armature will rotate in an anti-clockwise direction, or, if currents be passed in the opposite direction, both field and armature currents will be reversed (Fig. 125) and, consequently, rotation will take place in the same direction (*i.e.*, anti-clockwise).

In order to obtain rotation in a clockwise direction, either the field or the armature current, and not both, must be reversed. Hence, for a series dynamo to act as a series motor in the same direction, the field connections must be reversed (Fig. 126).

With a shunt dynamo, if Fig. 127 shows the direction of currents with clockwise rotation, we see from Fig. 128 that, if the armature be brought to rest and a P.D. applied at the brushes, either the field or the armature current is reversed, and not both, hence

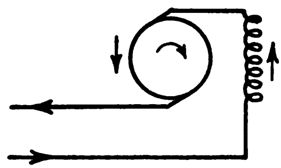

FIG. 124.—SERIES DYNAMO

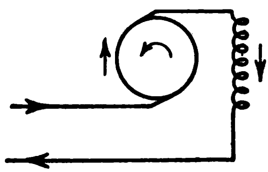

FIG. 125.—SERIES DYNAMO AS MOTOR

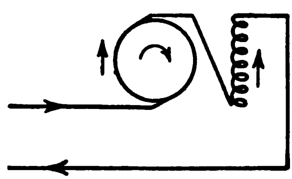

FIG. 126.—SERIES DYNAMO WITH FIELD REVERSED

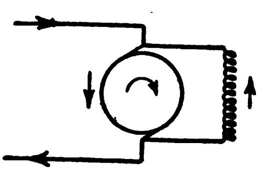

FIG. 127.—SHUNT DYNAMO

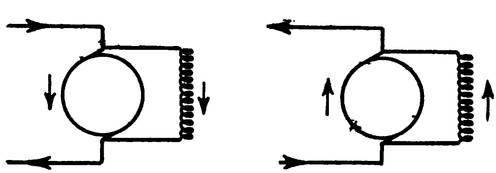

FIG. 128.—SHUNT DYNAMO AS MOTOR

Showing reversal of either field or armature current and not both

rotation will be clockwise (*i.e.*, the same as when acting as a dynamo, therefore no reversal of connections will be required.

Motors possess the same properties as dynamos with regard to commutation, but a consideration of Fig. 86 (Chap. VII) will show that, since either the field or the armature current is reversed, cross magnetization will take place in the opposite direction,

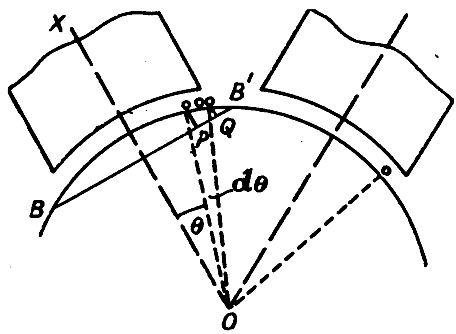

FIG. 129.—CALCULATION OF TORQUE OF MOTOR

hence the brushes must be given an angle of lag and not lead. This will, however, introduce a number of demagnetizing turns equal to $\frac{\lambda N}{\pi}$ exactly as before where λ is the angle of lag.

If the machine have interpoles, a similar consideration will show that they must have the polarity of those poles from which the armature is receding.

2. Torque.—As in the dynamo, consider a wire at an angle θ from OX. (Fig. 129.) The force acting upon it in a direction perpendicular to OX is $\frac{1}{10}BI_al$ dynes where I_a is the current in each conductor, and the component

in a direction tangential to the armature is $\frac{1}{10}BI_a l \cos\theta$ dynes. If there are σ wires per cm. of periphery, the force on the wires in a small arc $d\theta$ will be

$$\frac{1}{10}BI_a l\sigma r d\theta \cos\theta \text{ dynes.}$$

The turning moment about O, or the torque, will be

$$\frac{1}{10}BI_a l\sigma r^2 \cos\theta d\theta.$$

Hence, the torque per pole

$$= \frac{1}{10}I_a\sigma r \int_B^{B'} Blr \cos\theta d\theta \text{ dyne cms.}$$

As before $rd\theta \cos\theta = PQ$

$$\therefore \quad \int_B^{B} Blr \cos\theta d\theta = \text{the polar flux.}$$

Hence, the torque per pole

$$= \frac{1}{10}I_a\sigma r\Phi \text{ dyne cms.}$$

And if there are $2p$ poles, the total torque will be

$$\frac{2p}{10}I_a\sigma r\Phi \text{ dyne cms.}$$

Now the peripheral wires $= N = 2\pi r\sigma$

$$\therefore \text{ Total couple} = \frac{pI_a N\Phi}{10\pi}$$

Now, in the ring or lap winding, there are $2p$ paths for the current, and, in a wave winding, there are 2 paths.

Hence, if I be the current in amperes to or from the armature,

$$I = 2pI_a \text{ for ring or lap winding}$$
$$I = 2I_a \text{ for wave winding.}$$

$\therefore$ Torque $= \dfrac{N\Phi I}{20\pi}$ dyne cms. for ring or lap

and $\dfrac{pN\Phi I}{20\pi}$ dyne cms. for wave.

It has already been established that the E.M.F.

$E = \Phi Nn \times 10^{-8}$ volts for ring or lap

and $p\Phi Nn \times 10^{-8}$ volts for wave windings.

If, therefore, we write $\psi = [p]\Phi N \times 10^{-8}$ where $[p]$ means that p is excluded for ring or lap windings and included for wave windings, we may say

$$\text{Torque} = \frac{I}{20\pi}\psi \times 10^{8} \text{ dyne cms.}$$

$$= \cdot 117 I\psi \text{ lbs. ft.}$$

$$\text{and } E = \psi n \text{ volts.}$$

ψ is known as the induction factor.

It should be noted that the torque given above is the gross torque and includes that required to overcome friction, windage, etc.

In a motor the actual shaft torque will therefore be less than this value by the amount required to overcome the losses. Also, we see that, in a motor, the brush P.D. must equal the resistance drop in the armature (in a series machine, the armature + field coils) + the back E.M.F. generated.

§ 3. **Characteristics.**—The mechanical characteristics of a motor connect the shaft torque developed with the speed at constant P.D.

(*a*) SERIES MOTOR.—The shape of this curve for a series motor is given in Fig. 130.

Since the torque varies with the current, as the torque decreases the current will decrease, and therefore the field strength.

Hence, the speed must rise in order to produce the same back E.M.F. If, therefore, the machine became

unloaded, it would be liable to race to a dangerous speed.

Series motors possess the advantage of developing their maximum torque at the slowest speed, and are, consequently, almost invariably used for such purposes as traction work, where there is no likelihood of their becoming unloaded.

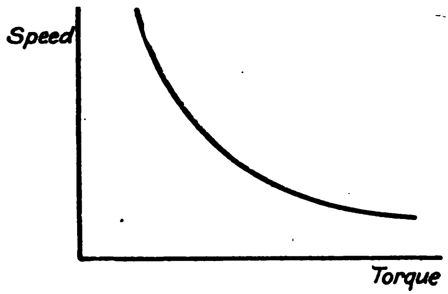

FIG. 130.—MECHANICAL CHARACTERISTIC OF SERIES MOTOR

The curvature of this characteristic is due to the shape of the *BH* curve. When the field is weak it increases rapidly with current, but much less rapidly as the higher portions of the *BH* curve are reached.

(*b*) SHUNT MOTOR.—Fig. 131 shows the shape of the shunt motor characteristic.

Since the field remains constant the speed will remain practically so.

As the torque increases the armature current increases, with a corresponding slight increase in voltage drop in the armature. This means a slight falling off in back E.M.F., and, consequently, a slight falling off of speed.

Shunt motors are used where it is desirable to maintain a fairly constant speed (*e.g.*, for driving the shafting in a machine shop and where a no load will not be dangerous).

(*c*) COMPOUND.—A compound machine combines the effect of (*a*) and (*b*) above, and is used in cases where a high torque is required at starting, as in the series motor; but where it is desired to avoid the danger of no load. This would be the case in motors required to drive planing machines, for example.

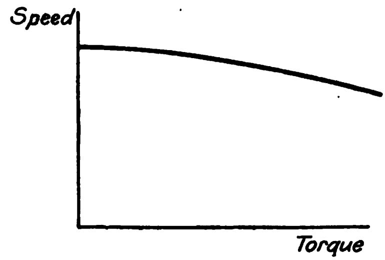

FIG. 131.—MECHANICAL CHARACTERISTIC OF SHUNT MOTOR

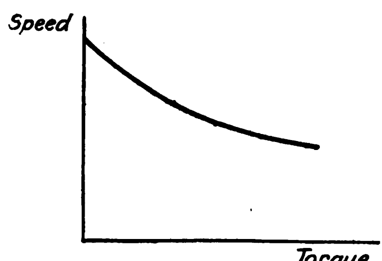

FIG. 132.—MECHANICAL CHARACTERISTIC OF COMPOUND MOTOR

In some machines the series field (which is comparatively weak) is arranged so as to oppose the shunt field. This is known as "differential" winding. The effect of this is to decrease the field in proportion as the load increases and thus to annul the droop in speed, due to voltage drop in the armature. It is possible to design such a machine so that it will maintain a constant speed over a very wide range of loads.

§ 4. **Losses.**—The losses which occur in motors are fundamentally the same as those in dynamos.

If a shunt machine be driven on no load, the current taken will be that required to overcome the various losses. If we call I_o that no load current, the lost torque will be $\cdot 117\, \psi I_o$ lbs. ft.*

Now, in a shunt motor, the speed and field are constant, consequently, we may say that, for a given armature current I, the useful torque developed will be $\cdot 117 \psi (I - I_o)$ lbs. ft.

Example

The armature of a 100 volt shunt motor takes 6 amps. on no load. The armature resistance is ·07 ohm, and the shunt field 40 ohms. Find the current taken and efficiency when driving a load of 10 H.P.

The power is $= 100\, I$ watts.

This is equal to the output + rotation losses + armature resistance loss + shunt resistance loss. Then, if I_a armature current and I_s = shunt current, we have

$$100(I_a + I_s) = 10 \times 746 + 100 \times 6 + I_a^2 \times \cdot 07 + 100 I_s$$

$$\text{Now} \qquad I_s = \frac{100}{40} = 2\cdot 5 \text{ amps.}$$

$$\therefore \quad 100\, I_a = 8060 + \cdot 07 I_a^2$$

$$\text{which gives} \quad I_a = 85\cdot 7 \text{ amps.}$$

$$\text{Hence,} \quad I = 88\cdot 2 \text{ amps.}$$

$$\text{Also efficiency } \eta = \frac{7460}{88\cdot 2 \times 100} = \frac{7460}{8820} = 84\cdot 6 \text{ per cent.}$$

* This neglects the copper losses in the armature at no load which, strictly speaking, should be subtracted in order to determine the rotational losses. In most practical cases, however, they are sufficiently small to be neglected as has been done in the example.

§ 5. Speed Variation.—The equation

$$E = [p]\Phi N n \times 10^{-8}$$

shows us that Φ and E are the two quantities by which we can vary n, the speed. In other words, we can either vary the field current or else the brush P.D., since this is practically the same as E. To vary the field in a series excited machine a variable resistance

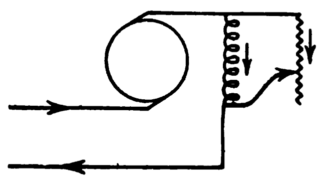

FIG. 133.—SERIES MOTOR WITH "DIVERTER"

may be shunted across the series field, and so divert some of the field current. (*See* Fig. 133.) This is known as a diverter.

Another method is to insert a variable resistance between the machine and the applied P.D. This has the effect of reducing the field, but reduces the P.D. to a greater extent, consequently the speed falls.

For a shunt machine, a variable resistance may be introduced in the field coils, so that Φ may be adjusted, or else an external resistance may, as in the series machine, be applied between the machine and the applied P.D., thus reducing the P.D.

The field variation method gives, of course, a lower ohmic loss than the other, but has the disadvantage that the machine must be made bigger if it has to develop the same torque at high speeds (*i.e.*, in a weak field), since we see, from the equation,

$$T = \frac{[p]N\Phi I}{20\pi}$$

that the current must be greater, and, consequently, the machine must be strong enough to carry it.

If there are interpoles, the speed may be varied by changing the position of the brushes. In a properly designed machine, with interpoles, sparkless commutation may be obtained for quite a wide range of positions of the brushes. Now, it was shown that giving the brushes a lag introduces demagnetizing turns, consequently, if the brushes are advanced, the number of these turns will be decreased, and if they are given a lead the turns then become magnetizing turns. Altering the position of the brushes, then, forms another means whereby the field strength, and therefore the speed, may be varied.

6. Motor Starters.—It has been shown that the P.D. across a motor has to overcome the back E.M.F. and also the voltage drop in the armature, or, symbolically, $V = E + I_a R_a$, where R_a is the armature resistance. Now E is, of course, in general very much greater than $I_a R_a$.

In the motor in § 4, $I_a R_a = 85{\cdot}7 \times {\cdot}07 = 6$ volts, and the back E.M.F. $E = 94$ volts.

At starting, since there is no rotation, there is no back E.M.F., and if the full P.D. were applied there would be a current of $\dfrac{100}{{\cdot}07} = 1444$ amps.

This would, of course, damage the armature and blow the fuzes. The P.D. must, therefore, be applied gradually and be increased slowly as the back E.M.F. increases.

The series motor switch is the simplest, and will be described first. Fig. 134 is a diagrammatic representation of it.

Only three steps of resistance have been shown, but the principle is unaffected by introducing more.

A is a metal arm pivoted at O. As it is moved

across the studs S, it cuts out more and more resistance as the speed of the motor rises.

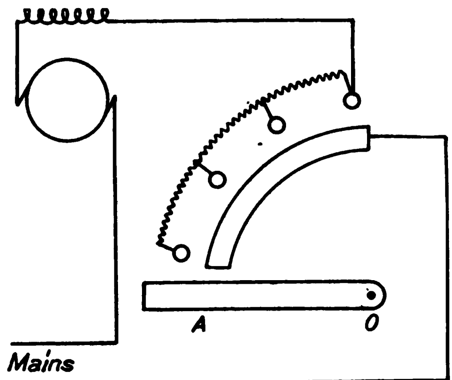

FIG. 134.—SERIES MOTOR STARTER

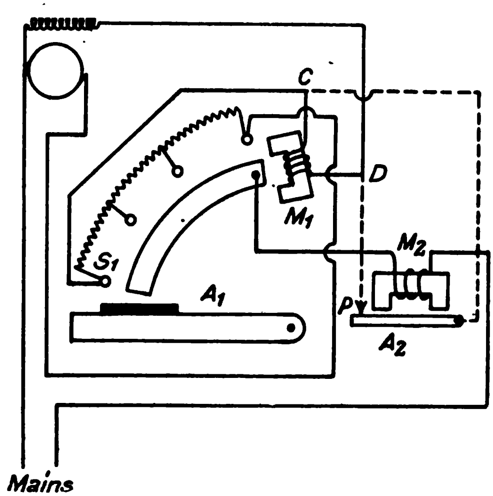

FIG. 135.—SHUNT MOTOR STARTER

For a shunt motor a slightly more complicated arrangement is adopted, as shown diagrammatically in Fig. 135.

Here it will be seen that the field coils are always in circuit with the armature. This is necessary, and the reason will be explained later. *M* is an electro-magnet round which the field current passes. It holds the arm A_1 in the on position against the pull of a spring.

M_2 is an electro-magnet round which the main armature current passes.

If the current exceeds a certain safe value it pulls the armature A_2 into contact with *P* against a small spring, thus short circuiting the current round M_1 at *C* and *D*. Then, M_1 having lost its magnetic pull, the lever A_1 flies into the "off" position.

M_1 is known as the "no volts," and M_2 as the "over load" release.

M_1 is necessary in order to prevent the possibility of the motor slowing down if the supply failed, and then receiving the full rush of current when supply is started again. With M_1 present, if the supply failed the switch lever would fly back and the motor would require to be re-started.

Although not shown in Fig. 134, there is usually a "no volts" coil and overload release attached to series starters, as well as shunt.

The field winding is connected to the first stud *S*, so that, at starting, there may be maximum field strength. When the machine is running, the resistances *S* are in series with the field but do not matter, since their resistance of a few ohms is small compared with that of the field coils.

When the machine is stopped, if the field circuit were broken the flux would die away rapidly and induce an exceedingly high E.M.F. in the many turns of the field coils, probably sufficient to break down

the insulation. As, however, the field circuit is always closed, this cannot happen. For one thing, the induced E.M.F. causes a current to flow which prevents the flux from dying away too rapidly (since induced E.M.F.'s are always in such a direction as to oppose the change which gives rise to them), and, for another thing, the armature is still rotating and its E.M.F. sends a current through the field in the same direction as before, also preventing the flux from collapsing too rapidly. (A few moments' consideration of § 1 will make it clear that the current is in the same direction as before.)

In a compound machine, the series coils are simply connected in series with the switch.

§ 7. Reversing Switches.—If the motor is required to run both ways, some form of reversing switch is required. Either the field or the armature current may be reversed, but the latter is more usual. If interpoles are used, they are in series with the armature current, so that when it is reversed their polarity is reversed, and they thus still fulfil the condition of being of the same polarity as the poles from which the armature is receding.

Fig. 136 shows the usual arrangement for a reversing switch, and it possesses the advantage that the current cannot be reversed without first being brought to the zero position. Only two steps of resistance R_1 and R_2 are shown.

The metal bars AA, electrically insulated from each other, make contact with the strips B, C or C_1, D or D_1, and the studs S. It will be seen that the direction of the field is constant, and that the field coils are short-circuited in the "off" position by AA. As in the shunt motor switch described in the preceding section, AA is held in the "on" position by a "no volt" release magnet (not shown).

With all forms of starters it is, of course, necessary that the various steps of resistance should not be cut out too rapidly. The main ammeter should be watched so that contact is not made with another notch until the motor has speeded up sufficiently so as to cause the current to fall to its correct amount each time. Of course, if a continuous variation of

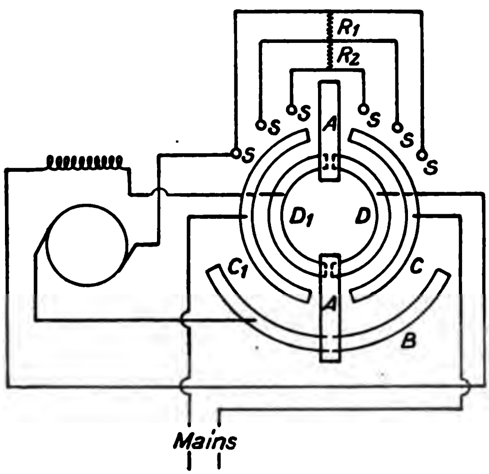

FIG. 136.—SHUNT MOTOR REVERSING STARTER

resistance were possible, the current could be kept constant by reducing the resistance gradually as the speed and, therefore, the back E.M.F. rose. As the process must be in steps, however, the current is bound to jump from its normal to a higher value, and then gradually fall to the normal again as the speed and, therefore, the back E.M.F. increases.

In order to prevent the resistance from being cut out too rapidly, a device is sometimes introduced, usually on the larger types of machines, whereby the

moving arm is attached to a worm wheel with which a worm is geared and motion is obtained by means of a crank handle at the end of the worm spindle.

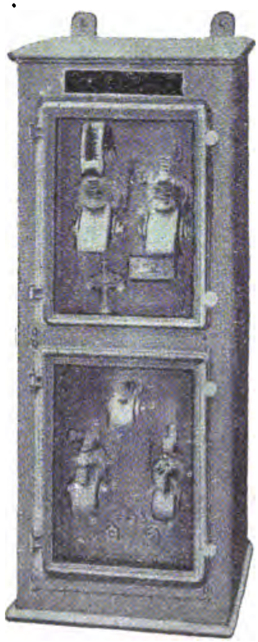

FIG. 137.—METROPOLITAN VICKERS AUTOMATIC CONTROL

By this means, rapid motion of the contact arm becomes impossible. With this arrangement there must, of course, be some form of release, so that the arm may fly back to the "off" position rapidly when the "no volts" or over-load coil is actuated.

§ 8. Automatic Controls.—It is sometimes desirable where large motors are used to have a means whereby the motor may be started and stopped automatically, the various steps of resistance being cut out at their correct intervals of time, and, for this purpose, some form of "contactor" control, as it is called, is adopted. The following is a brief description of one of the simplest types of these devices manufactured by Messrs. Metropolitan-Vickers, of Manchester. It is designed to carry 125 amps.

Figs. 137 and 138 show a photograph and diagram of connections of the apparatus; 1, 2, and 3 are the main contactors, normally held apart by springs, but contact being made when the coils *OC* 1, *OC* 2,

and OC 3 are excited. AR_1 and AR_2 are known as accelerating relays, the plungers of which tend to make contact at X and Y under the action of light springs, whose tension may be adjusted. By a simple mechanical arrangement the plunger of AR_2 is prevented from making contact at Y until after contactor 2 is closed. OR is the overload relay by means of which contact is broken at Z if the main current becomes too

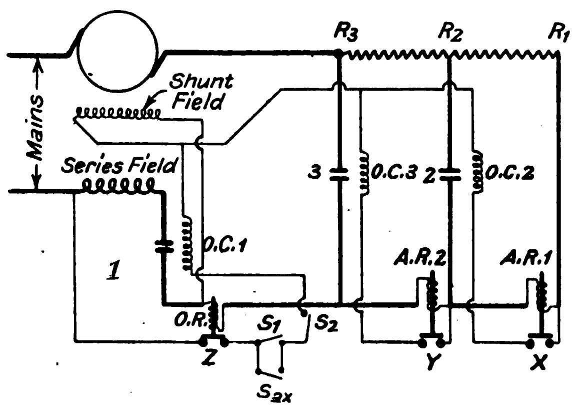

FIG. 138.—CONNECTIONS FOR COMPOUND MOTOR AUTOMATIC STARTER

high. S_1 and S_2 are starting and stopping switches, and S_{ax} is an auxiliary starting switch. R_1, R_2, and R_3 correspond to the contacts of a three-step rheostat.

In operation, the main switch at the bus bars is closed and the contact is made either at S_1 or S_{ax}. This causes a current to pass through OC 1 which closes 1. Both fields then become excited, and a current passes into the armature through AR_2 and AR_1, and the full rheostat resistance. As the motor speeds up and the back E.M.F. rises, the current falls until it

is insufficient to retain the plunger of AR_1 against the spring, and contact is made at X. (It will be remembered that AR_2 is retained mechanically by means of 2.) Current is now able to pass through OC 2 and contact is made at 2, cutting out the resistance between R_1 and R_2. The main current at once rises, and is now sufficient to hold the plunger of AR_2 against the spring.

(*Note.*—It is essential that contact be made at 2 before the mechanical retention of AR_2 is disturbed. Means are provided whereby this may be adjusted.)

Acceleration continues until the current once more falls, enabling the spring of AR_2 to assert itself and contact is made at Y.

A current can now pass through OC 3, causing contact to be made at 3, thus cutting out the second and final step of resistance. OR is known as the over-load relay. If the current taken by the motor becomes too high, the plunger is withdrawn, contact is broken at 2, and the motor stops; all contacts 1, 2, and 3 being at the same time broken.

An instance of the use of the device is in a water accumulator working like a gasometer.

As water is used, the top section of the accumulator sinks, until, at a specified point a mechanical device causes contact to be made at S_1, thus starting up the motor which drives the water pump. When the accumulator is nearly full it trips the contact S_2, thus stopping the motor; these operations, of course, being entirely automatic in their action. S_{ax} is connected to a switch so that the motor may be started at will if necessary.

There are many other more complicated types of this apparatus containing more steps of resistance, or including reversing connections, etc., but they all depend upon the principles described above.

It will be noted that the shunt field is always in circuit through the armature.

§ 9. Calculation of Starter Resistances.—The factors which determine the amount of resistance in each step are the working current of the armature, and the safe maximum current which may be allowed to pass through the armature. Sometimes the difference between these is so great however, that if they alone were considered, there would be quite a large current rush as each section of resistance is cut out, and this might result in a fluctuation of current in other loads, such as lamps, etc., connected to the same source of supply.

In this case, it is the maximum permissible current "rush" which determines the various resistances.

In the series motor the field varies with the current.

Suppose the working value of current is I_1, and the safe maximum is I_2, and that the field strength corresponding to these currents are Φ_1 and Φ_2 respectively.

Let R_1 be the total resistance of the rheostat, *including* R_a, the resistance of the motor armature and field. On making the first contact there is no rotation, and, consequently, no back E.M.F. The value of R_1 is therefore given by $R_1 = \dfrac{V}{I_2}$, where V is the applied P.D.

The arm is left on this stud until the speed rises sufficiently to cause the current to fall to I_2. Call the speed n_1,

(*a*) On leaving first stud, back E.M.F.

$$= \Phi_1 N n_1 \,.\, 10^{-8} \text{ and } I_1 = \frac{V - \Phi_1 N n_1 10^{-8}}{R_1}$$

Let the second stud reduce the total resistance to R_2.

(*b*) On touching second stud, back E.M.F.

$$= \Phi_2 N n_1 \,.\, 10^{-8} \text{ and } I_2 = \frac{V - \Phi_2 N n_1 10^{-8}}{R_2}$$

Suppose the speed falls to n_2.

(*c*) On leaving second stud, back E.M.F.

$$= \Phi_1 N n_2 10^{-8} \text{ and } I \;= \frac{V - \Phi_1 N n_2 10^{-8}}{R_2}$$

(*d*) On touching third stud back E.M.F.

$$= \Phi_2 N n_2 10^{-8} \text{ and } I_2 = \frac{V - \Phi_2 N n_2 10^{-8}}{R_3}$$

Subtracting (*c*) from (*a*) and (*d*) from (*b*), we have

$$I_1(R_1 - R_2) = \Phi_1 N 10^{-8}(n_2 - n_1)$$

and $$I_2(R_2 - R_3) = \Phi_2 N 10^{-8}(n_2 - n_1)$$

$$\therefore \quad \frac{R_1 - R_2}{R_2 - R_3} = \frac{\Phi_1 I_2}{\Phi_2 I_1} = \text{const.}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \text{etc.} = \frac{\Phi_1 I_2}{\Phi_2 I_1} = \text{const.} \qquad (27)$$

And, if R_a is the resistance of the armature and field coils and there are n steps,

$$\frac{R_1}{R_a} = \left(\frac{\Phi_1 I_2}{\Phi_2 I_1}\right)^n$$

$$\text{or } \log R_1 - \log R_a = n\,(\log \Phi_1 I_2 - \log \Phi_2 I_1)\,. \qquad (28)$$

Hence, from (27) and (28) we can calculate the number of steps required, and also the reduction ratio of each step.

EXAMPLE

A 15 H.P. series motor is run off 400 volt mains. The armature resistance is 1 ohm. Its maximum permissible current is 40 amps. and the working current is 28 amps. The field strengths at these currents are $1{\cdot}26 \times 10^6$ and $1{\cdot}00 \times 10^6$ respectively.

Find the number of steps in the rheostat and also the reduction ratio of each step.

At starting, we have

$$R_1 = \frac{400}{40} = 10 \text{ ohms.}$$

Hence, total rheostat resistance is $10 - 1 = 9$ ohms.

$$\text{Now} \quad \frac{10}{1} = \left(\frac{1{\cdot}00 \times 10^6 \times 40}{1{\cdot}26 \times 10^6 \times 28}\right)^n$$

$$= \left(\frac{40}{35{\cdot}2}\right)^n = 1{\cdot}135^n$$

$$\therefore n = \frac{\log 10}{\log 1{\cdot}135} = \frac{1}{{\cdot}065}$$

$$= 15{\cdot}4$$

or 16, say, since the number must, of course, be an integer, also the reduction factor

$$= \frac{\Phi_1 I_2}{\Phi_2 I_1} = 1{\cdot}135$$

This number 16 is high and is due to the rather large variation in field with current.

If the iron were fairly near saturation, there would not be such a large variation in field and the number n would be less.

In a shunt motor, a field is, of course, constant. consequently,

$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \ldots \ldots \frac{I_2}{I_1}$$

$$\text{and} \quad \frac{R_1}{R_a} = \left(\frac{I_2}{I_1}\right)^n$$

Hence, if the motor above were shunt wound we should have as before

$$R_1 = 10$$

$$\text{and} \quad 10 = \left(\frac{40}{28}\right)^n = (1{\cdot}46)^n$$

$$\therefore n = \frac{1}{\log 146} = \frac{1}{\cdot 155} = 6{\cdot}4 \text{ or } 7, \text{ say.}$$

The reduction factor is $I_2/I_1 = 1{\cdot}46$.

If the value I_2 is greater than the permissible current rush there must be extra steps put in before the first

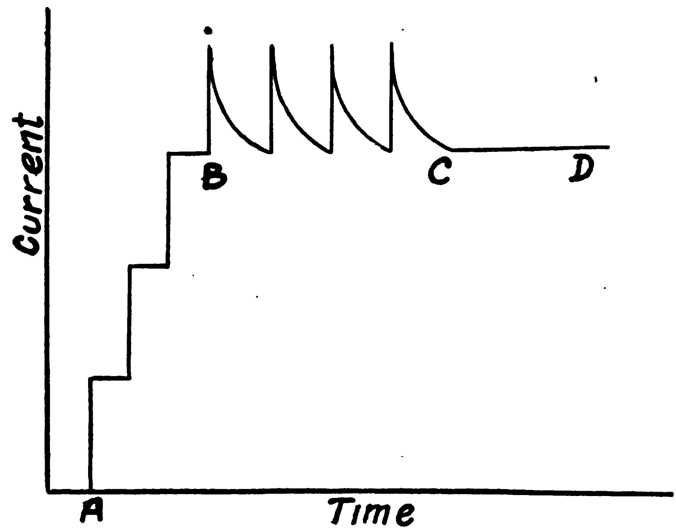

FIG. 139.—CURRENT VARIATION WHEN STARTING A MOTOR

AB. Steps to sub-divide current rush
BC. Further steps, motor accelerating
CD. Steady maximum speed

in order to sub-divide this current rush to its correct value. The variation of current with time would then be as shown in Fig. 139.

§ 10. **The Motor Generator.**—Occasionally it is desired to utilize power at a different voltage from that of the supply mains. In these cases it is usual to have a motor and generator coupled together, the

motor to be run from the supply mains and the generator wound so as to give the required voltage. Such sets are usually used when the voltage required is less than that of supply.

If a greater voltage is required, it is more usual to use an auxiliary machine or booster in series with supply, to give the additional voltage required.

It would be possible to place both windings of the motor generator set on the same armature with two sets of commutator segments and brushes, thus needing only one machine, but this is not usually done in practice, since it is not practicable to regulate the P.D. produced by varying the field since a reduction of field will increase the speed and thus leave the E.M.F. unchanged.

The power delivered by the generator in the combined set is, of course, equal to that supplied to the motor, less the losses in the two machines.

§ 11. The C.M.B. Converter.—This is another machine for delivering power at a reduced voltage. In the main, it consists in tapping the armature of a motor at two points at a less distance apart than the polar pitch. A complete study of the machine is beyond the scope of this book, but it will be obvious from a study of Fig. 74 (Chap. VI) that if an additional brush be placed at some point between B and B', the integration of E.M.F.'s to this brush will give a lower value than that from B to B'. In practice, this machine presents many difficulties in design, the main one, perhaps, being that the poles require to be of special shape, usually with a radial slot cut in at the points opposite where the auxiliary tappings are made. This is necessary in order to provide that the coils, undergoing commutation at the auxiliary brushes, shall not have excessive E.M.F.'s induced in them, as it would lead to sparking.

Examples

(1) A 6 pole lap wound machine carries 400 wires. The flux per pole is $1{\cdot}54 \times 10^6$ C.G.S. lines, and the armature current is 25 amps. Find the gross torque developed in lbs.-feet units. The applied P.D. is 100 volts, and the armature resistance ·1 ohm.

If the field be shunt wound, find the speed at which the motor will run.

(2) Explain how and why the speed of a shunt motor varies when its field current is varied. Assuming the efficiency to be 85% and that copper losses are 40% of the total losses, work out the change in field current necessary to give a 5 per cent. increase in the speed of a 5 B.H.P. 200 volt shunt motor running on full load. (Torque constant.)

(*Ordnance College, Woolwich, Advanced Class*, 1919.)

(3) A series motor of resistance 1·5 ohm is connected to a P.D. of 400 volts. When running at 360 revs. per minute it takes 30 amps. to produce a certain gross torque. If the torque is reduced by 50 per cent., and the current to 20 amps., find the speed.

CHAPTER XII

TESTS

§ 1. In the two preceding chapters we have treated the operation of a machine from a theoretical point of view and obtained formulae, empirical and otherwise, which may be used by a designer to pre-determine the efficiency of his machine. In the present chapter we propose to treat the testing of dynamos and motors from a practical point of view.

Machines are always tested for the following—

(1) Efficiency.
(2) Heating.
(3) Regulation.

(1) Efficiency.—This may be determined: (*a*) directly, where it is possible to measure the intake and the output; (*b*) indirectly, where either the output or intake and the losses are measured. Then the efficiency (η) is given by

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$\text{or} \qquad = \frac{\text{intake} - \text{losses}}{\text{intake}}$$

(2) (*c*) Regenerative Methods. (*See* later.)

§ 2. The first method (*a*) is only suitable for small machines of low power (up to about 5 H.P.) and small efficiency, but for large and efficient machines this method is neither suitable nor sufficiently accurate.

There are various direct methods of which the following is typical. If the machine is a dynamo, we may drive it by means of a standardized motor; that is, a motor whose efficiency curve is known

accurately. Therefore, knowing the intake of the motor, measured electrically, say $E_M I_M$, and its efficiency for this load η_M, its output will be given by $\eta_M E_M I_M$. We will suppose that the motor is coupled direct to the dynamo, therefore the intake of the dynamo will be $\eta_M E_M I_M$. If the output of the dynamo

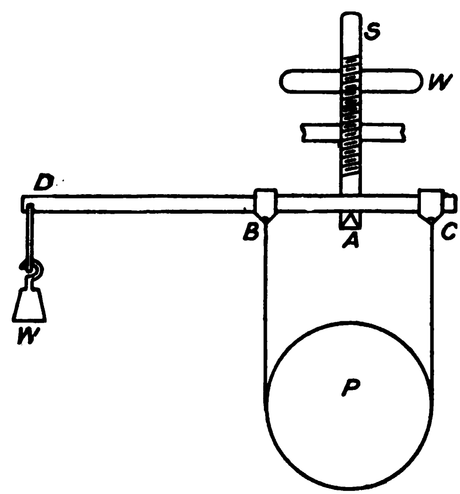

FIG. 140.—SOAMES' BRAKE

is $E_D I_D$, which may be measured easily, the efficiency of the machine is

$$\eta_D = \frac{E_D I_D}{\eta_M E_M I_M}$$

§ 3. **Soames' Brake.**—This is a mechanical device for absorbing the output of a motor by means of friction.

A steel bar is pivoted at A. From the points B and C a strong belt (usually made of hemp) is suspended and passes round the pulley of the machine P. The

belt may be tightened on the pulley by means of a screw S which is elevated or lowered by the wheel W. The load is applied by a variable weight W, attached to the end (D) of the steel bar. Let l represent the distance DA in feet and r the distance AC, which is adjusted so that it is equal to the radius of the pulley. Let T represent the torque developed by the motor when running at a speed of n revs. per sec.

Then the work done per second $= 2\pi nT$

The torque T may be measured, being equal to Wl.

Hence the work done per second $= 2\pi nWl$, and the output of the motor in horse-power $= \dfrac{2\pi nWl}{550}$

The intake of the motor may be measured electrically by a voltmeter and an ammeter, and, expressed in horse-power, will be $\dfrac{EI}{746}$. Therefore the efficiency of the machine is

$$\eta = \frac{\dfrac{2\pi nWl}{550}}{\dfrac{EI}{746}}$$

This method is only suitable for small machines of output up to 4 or 5 kws.

§ 4. **Eddy-Current Brakes.**—This is a similar but more accurate method of measuring the output of a motor. One or more thick copper discs are coupled to the shaft of the machine. These are rotated between the poles of electro-magnets, which are attached to a pivoted lever with one long arm, from the end of which weights may be suspended. By their rotation in the magnetic field, eddy currents are induced in the discs, which exert a torque on the electro-magnets. The magnets will therefore be dragged

round in the direction of rotation of the discs. They may be held stationary by applying an opposing torque by means of a weight attached to the long arm of the lever.

This type of brake has a considerable advantage over the preceding type, in that the resistance is more

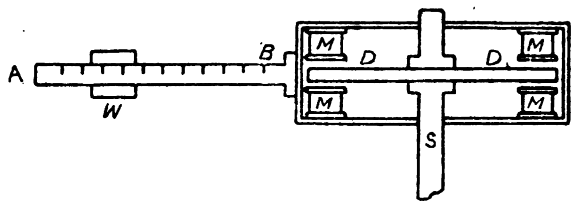

FIG. 141.—EDDY CURRENT BRAKE

S. Shaft
DD. Copper disc
MM. Electro magnets
W. Sliding weight
AB. Graduated arm

uniform and errors due to heating the pulley wheel by the belt are eliminated.

For testing machines of larger power an armature replaces the disc, and a pivoted field system the electro-magnets and lever.

§ 5. Field's Test for Two Exactly Similar Series Motors.—This method is used in testing tramway motors. The machines are mechanically coupled so that their speeds, and therefore, since the machines are exactly alike, their friction losses will be equal. One machine is driven as a motor and the energy supplied measured electrically. Let $E_1 I_1$ represent the voltage and current required. The other machine is driven as a dynamo and supplies a load current I_2 at a voltage E_2. The machines are connected so that the field windings of the two machines are in series with the motor. Therefore the field strengths will be equal except

for armature reaction, and hence the iron losses will be approximately the same.

Let R_a represent the armature resistance of each machine.

Let R_F represent the field resistance of each machine.

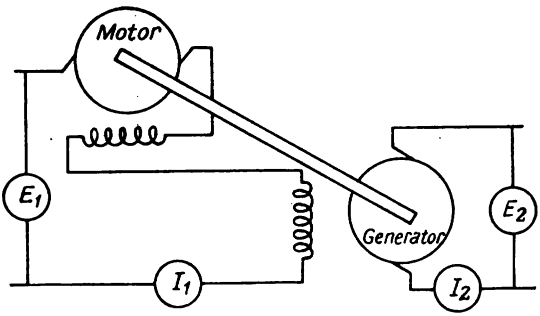

FIG. 142.—FIELD'S TEST FOR TWO SIMILAR SERIES MOTORS

The total loss in the two machines

$$= E_1I_1 - E_2I_2 = W_1$$

The total copper loss is given by

$$(R_a + 2R_F)I_1^2 + R_aI_2^2 = W_2$$

The iron and friction losses in the two machines is the difference of W_1 and W_2

$$W = W_1 - W_2$$

These are approximately equal in the two machines, therefore the efficiency of the motor is given by

$$\eta = \frac{E_1I_1 - \frac{W}{2} - R_aI_1^2 - R_f I_1^2}{E_1I_1}$$

§ 6. **Swinburne's Test.**—This is an indirect method of testing in which the losses are measured. It

consists in running the machine as a motor by applying a P.D. to its terminals equal to that of full load. The normal speed is obtained by varying the current in the field windings. If I is the armature current and E the applied voltage when the machine is running under these conditions, the product EI represents the power lost. The copper losses may be calculated in the usual manner. Let these be equal to W_1.

Then $EI - W_1$ will be equal to the friction and iron losses. Let

$$EI - W_1 = W.$$

The method assumes that the latter are constant at all loads. Therefore, knowing the "hot" resistances of the field and armature windings, the efficiency of the machine may be calculated for any load current. Thus, for any load I,

$$\text{Efficiency} = \frac{EI - W - \text{Copper loss}}{EI}$$

The assumption that the iron and friction losses are constant at all loads, though convenient for many practical purposes, is not strictly accurate. Since, when the armature is carrying a considerable load current, armature reaction causes a distortion of the field, which produces an iron loss greater than that when the armature is carrying a negligible current, as in the case of the above test.

§ 7. **Regenerative Methods.**—The preceding tests have the disadvantage that, either the efficiency is not measured under working conditions, or the test is only applicable to machines of small output and low efficiency. The following method, due to Hopkinson, overcomes these difficulties. It may be applied to machines of any size provided that two identical machines are available. Further, little outside power is required. This is a considerable advantage in testing

machines of large output, since the power required to run them at full load may not be available. The principle of the test is as follows. Suppose that two identical machines are coupled on to a shaft which is driven by some external force. Also, suppose that the two machines are connected electrically. If the field excitation of one, say (A), is increased, it will generate a larger E.M.F. than the other (B), so that current will flow from A to B, and B will therefore be driven as a motor.

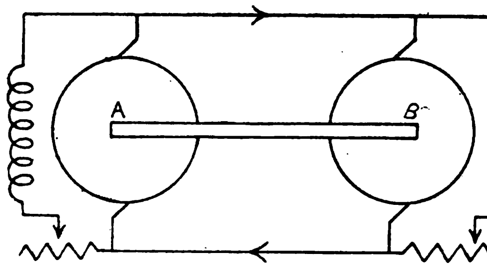

FIG. 143.—HOPKINSON TEST

If there were no losses in the two machines then, once started, A would drive B without any power being supplied by the external source. Practically, power is supplied, and the amount will be equal to the losses in the two machines. If the machines are driven at normal speed, their fields unexcited, the power supplied represents the friction and windage losses. Under unequal excitation the power circulates from one machine to the other.

In the test as originally suggested by Hopkinson, the external power was supplied mechanically and measured by means of a transmission dynamometer. Let W represent the power supplied.

Then, for machine A generating an E.M.F. $= E$,

and having a load current I, the output is EI. Assuming that the losses are equal in the two machines, the intake is $EI + \frac{W}{2}$.

Therefore the efficiency of A is given by

$$\eta_A = \frac{EI}{EI + \frac{W}{2}}$$

Similarly, the efficiency of B is given by

$$\eta_B = \frac{EI - \frac{W}{2}}{EI}$$

The mechanical power supplied is not easily measured, and an improvement on the above test was suggested by Kapp. Instead of mechanical power being used to drive the two machines, electric power from a dynamo or cells may be employed. The power is thus measured with considerably greater accuracy and convenience. Fig. 144 represents diagrammatically the skeleton of the connections.

The battery, or generator, supplying the power is maintained at a constant voltage E, the full load voltage of the machine under test. Let i be the total current supplied to the set. This will be equal to the difference of the two currents I_A and I_B, measured by the ammeters in the circuits of the machines A and B respectively,

$$i = I_B - I_A .$$

Machine A will be acting as the generator and B as the motor.

Assuming that the losses in the two machines are equal, the total loss in each is

$$\frac{Ei}{2} = \frac{E(I_B - I_A)}{2}$$

The efficiency of the generator A is given by

$$\eta_A = \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{EI_A}{E\left(I_A - \frac{i}{2}\right)}$$

and since $i = I_B - I_A$

$$\eta_A = \frac{2I_A}{I_A + I_B}$$

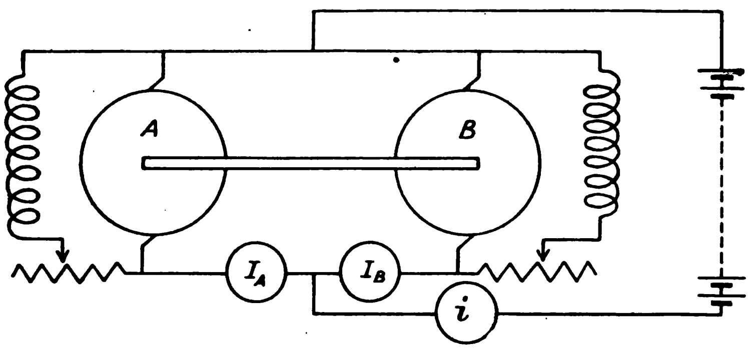

Fig. 144.—Hopkinson Test

Similarly, the efficiency of the machine B which is driven as a motor is

$$\eta_B = \frac{I_A + I_B}{2I_B}$$

§ 8. The Series Hopkinson Test.—Another variation of the Hopkinson test, known as the series test, is as follows—

The two machines are connected in series and coupled to the same shaft.

In the same circuit a dynamo is placed, as shown in Fig. 145. This machine is capable of supplying the full load current of the two machines at a small voltage. For convenience, the field windings of the machine under test are connected to mains which are maintained at the full load voltage of the two machines.

Let E_1 represent the voltage generated by A.

Let E_2 represent the voltage applied to the machine B which is running as a motor.

If E is the voltage generated by the auxiliary machine C, then the power it supplies to the circuit is EI, where I is the current flowing, EI will be equal

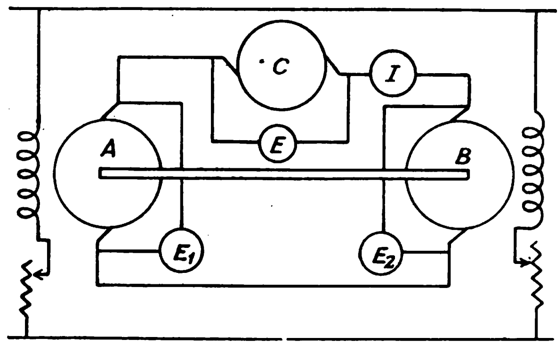

FIG. 145.—THE SERIES HOPKINSON TEST

to the combined losses of the machines A and B. Therefore, if the two machines are identical, the losses in each may be taken as $\frac{EI}{2}$

The output of the generator (A) may be measured by means of a voltmeter and an ammeter, and is equal to E_1I. The losses in A will be $\frac{EI}{2}$ + Field loss. Let the field loss be ${}_FW_A$. This is measured in the usual manner.

Then the efficiency of A for any load I is given by

$$\eta_A = \frac{E_1I}{E_1I + \frac{EI}{2} + {}_AW_F}$$

Similarly, the efficiency of the machine (B) which is driven as a motor is

$$\eta_B = \frac{E_2I - \frac{EI}{2} - {}_BW_F}{E_2I}$$

§ 9. **Heating or Temperature Test.**—This test may be carried out in a number of ways, depending on the output and the type of machine to be tested. We will take as a typical case the compound wound generator,

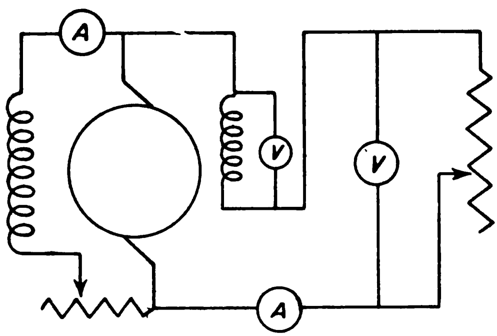

FIG. 146.—CONNECTIONS FOR HEATING AND REGULATION TESTS

so that, with a few modifications, the method may be applied to other machines.

The generator is direct coupled to a motor or engine capable of supplying at least 50 per cent. more power than the generator, arrangements being made to run the latter at its rated speed. The loading may be accomplished by means of a water rheostat. The connections will be those shown in Fig. 146.

Ammeters are placed in the shunt and main circuits and voltmeters across the series coil and load resistance. The machine is then run up to a speed and the full

voltage obtained by adjusting the field rheostat. Next, the full load current is obtained by cutting out the resistance. The readings on the various instruments are taken from time to time during the run. The rise in temperature of the various coils may be measured by their rise in resistance. Tables and figures showing the maximum permissible rise are given in Fowler's *Electrical Engineer's Pocket Book.*

§ **10. Regulation.**—This test, as the name implies, is made to determine the behaviour of the machine between no load and full load. The details are somewhat different for the various machines, but if we take typical examples of compound and shunt generators, similar tests may be applied to the remaining types.

A compound machine (*see* Chap. X) may be designed to give a flat or rising characteristic, according to the degree of compounding. In this test the field is set at no load, so that the generator gives its full voltage when running at its rated speed. The load is now put on and readings taken of the various instruments exactly as in the temperature test, for which the connections are identical. Regulation curves may then be plotted showing the relation between terminal volts and load current.

For a shunt machine the test is carried out in an identical manner, with the exception that the shunt rheostat is adjusted from time to time so as to increase the field current and keep the voltage constant as the load increases.

Examples

(1) The following are data taken from a Soames brake test. Applied voltage 94, load current 42 amps., speed 835 r.p.m., length of lever = 1·31 ft., weight attached to lever = 20 lbs. Find the efficiency of the motor.

(2) A small motor rated at 1 H.P., 100 volts, 1000 r.p.m., is tested as follows—

(*a*) Motor running light: Voltage 100, armature current 1·6 amps., shunt current 1·1 amps.

(*b*) Motor loaded: Armature current 12 amps.

The resistance of the armature is ·673 ohms. Find the efficiency of the motor under the above load.

(3) Calculate the electrical efficiency of the shunt dynamo from the following data—

Terminal voltage = 80; current supplied to external circuit = 58 amps.; armature resistance ·06 ohms; resistance of field coils 14·6 ohms.

ANSWERS TO EXAMPLES

CORRIGENDA

ANSWERS TO EXAMPLES

Page 215. Chapter I
(1) for ·022 read ·056.
" " Chapter II
(4) for ·315 read ·00315.
" 216. Chapter X should read Chapter XI.
" " Chapter XI should read Chapter XII.
" " (2) for 67·2 read 72.

(5297)

meter.

52·2 volts.

(2) 8·85 volts; 16,200 C.G.S. lines.
(3) 171 volts.

ANSWERS TO EXAMPLES

CHAPTER I.

(1) ·022 dynes.
(2) 11·2 dynes ; 6·1 dynes.
(3) 5.

CHAPTER II.

(1) ·038 coulombs.
(2) 23·7 volts.
(3) ·45 amps.
(4) ·315 volts.

CHAPTER III.

(1) 166 watts.
(2) 1,655 amp. turns.
(3) 9·45 amps.
(4) 603, 960 amp. turns.

CHAPTER IV.

(1) Shunt resistance = $\frac{1}{9}$ that of ammeter.
(2) Resistance = ·005 ohm.
(3) Shunt resistance = ·003 ohm.

CHAPTER V.

(1) ·1 ohm ; 22 amps. ; 38·2 volts ; 52·2 volts.
(2) 88% ; 75% ; 165 amp. hours.

CHAPTER VI.

(1) Lap ; $1{\cdot}76 \times 10^6$ C.G.S. lines.
(2) 8·85 volts ; 16,200 C.G.S. lines.
(3) 171 volts.

CHAPTER VII.

(1) 106 volts ; 3·33 amps. ; 3·43 amps.
(2) ·76 cm. ; 1,110 $cms.^2$
(3) 9×10^6 C.G.S. lines ; 5,610.

CHAPTER VIII.

(1) ·8 volt.
(2) ·78 volt.
(3) 720 amp. turns.

CHAPTER IX.

(1) 74%.
(2) 85·4%.
(3) (*a*) 144 watts ; (*b*) 297 watts.

CHAPTER X.

(1) 18·2 lbs. ft. ; 990 revs. per min.
(2) 5·3%.
(3) 498 revs. per min.

CHAPTER XI.

(1) 78·2%.
(2) 67·2%.
(3) 88·4%.

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